Last Name: ___________ First Name: _____________ Section 141 - _____ UIN _____________

Counting Part 2

Part I

1. **a.** Using one RED, one BLUE, one GREEN, and one ORANGE disc, list all the ways you can choose a set of 2 discs if the order in which you choose them *does not* matter, i.e., choosing RB is the same as choosing BR. The list of 6 has been started for you.

RB, RG, RO, ______, ______, ______

b. There is a total of _______ ways to choose 2 of 4 discs, where order doesn't matter.
When order mattered, there were $P(4, 2) = 12$ ways to arrange them. Now that order doesn't matter, there are only 6 ways to choose the set of two.

2. **a.** If you had 5 discs of different colors (R, B, G, O, P), list all the ways that you could choose a set of 4 in which the order that the discs are chosen doesn't matter, i.e., RBGO is the same as GRBO.

____

b. There is a total of _______ ways to choose 4 of 5 discs, where order doesn't matter.

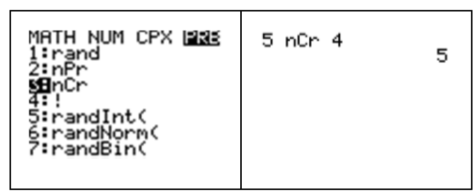
If order mattered, there would be $P(5, 4) = 120$ ways to arrange these discs. Now that order doesn't matter, there are only 5 ways to choose the set of four.

A selection of objects, where order does NOT matter, is called a *combination*. The number of combinations of *n* objects taken *r* at a time is

$$C(n,r) = \frac{n!}{r!(n-r)!}$$

Using this notation, the number of ways to choose 2 of the 4 colored discs in **1.** is $C(4, 2)$ and the number of ways to choose 4 of 5 colored discs in **2.** is $C(5, 4)$.

The formula above may be used, but there is a built-in function on your calculator that will calculate the numerical value of $C(n, r)$. Enter the value of n, press MATH, cursor right to the PRB menu, choose option 3, enter the value of r, and press ENTER.



Verify your answers to **1.** and **2.** on your calculator.

3. Find the number of ways to choose 6 crayons from a box of 24 crayons. *C*(____, ____) = ________

4. Find the number of ways to choose 3 pens and 4 pencils from a drawer containing 8 pens and 12 pencils.

a. In how many ways can the 3 pens be chosen from the 8 in the drawer? *C*(____, ____) = __________

b. In how many ways can the 4 pencils be chosen from the 12 in the drawer? *C*(____, ____) =_________

c. In how many ways can **both** the pens **and** pencils be chosen?
Hint: Use the Multiplication Principle.

___________________ x ___________________ = ____________________
\# of ways to choose pens   \# of ways to choose pencils

Some examples of other situations involving combinations are

- **Choosing a 3-member committee from a group of 40 people (order does not matter since the 3 people are being chosen for the same job);**
- **Choosing a committee of 3 males and 1 female from a club with 7 male and 8 female members;**
- **Selecting a sample of 8 light bulbs in which 2 are defective from a box of 30 bulbs that contains 10 defectives.**

Part II

1. In how many ways can two sets of 5 dominoes be selected from a box of 28 dominoes?

$C($ ____, ____ $)$	x	$C($ ____, ____ $)$	=	______________________
# of ways to choose 5 dominoes from 28 for the 1st hand		# of ways to choose 5 dominoes *from the remaining 23 dominoes* for the 2nd hand		

Note: $C(28, 10)$ is incorrect since it would give the number of ways to select 10 dominoes from the set of 28, but does not indicate which 5 dominoes go to which hand.

2. Suppose you buy 9 different canned goods at the grocery store and you want to arrange them in your pantry. The bottom shelf fits 9 cans and the top shelf fits 6 cans.

a. In how many ways can all 9 be arranged on the bottom shelf? ______________________

b. In how many ways can 6 of the 9 cans be arranged on the top shelf? ______________________

c. Of the 9 cans, 2 are canned vegetables, 3 are canned fruit, and 4 are soups. If you want to arrange all 9 cans on the bottom shelf *by type*, in how many ways can this be done?

One possible arrangement of the 9 cans is the following: $\underbrace{V_1V_2}\,\underbrace{F_1F_2F_3}\,\underbrace{S_1S_2S_3S_4}$

To create other arrangements, you can rearrange the order in which the groups appear and you can rearrange the cans within each group. For example, another possible arrangement is $\underbrace{S_3S_2S_4S_1}\,\underbrace{V_2V_1}\,\underbrace{F_1F_3F_2}$. The total number of arrangements is found as follows:

________	x	________	x	________	x	________	=	______________
# of ways to arrange the 3 groups		# of ways to arrange the 2 vegetable cans		# of ways to arrange the 3 fruit cans		# of ways to arrange the 4 soup cans		total # of ways to arrange cans by type

3. At a fruit stand there are 15 watermelons, of which 6 are ripe. A sample of 7 watermelons is selected at random. How many samples would have exactly 5 ripe watermelons?

a. If 6 of the watermelons are ripe, then _____ are not ripe.

15 *total*

6 *ripe*
___ *not ripe*

b. In the sample of 7, if you want to select exactly 5 ripe watermelons, then you will have exactly ______ unripe watermelons in that sample.

c. The 5 ripe watermelons in your sample must be chosen from the 6 ripe watermelons at the fruit stand. Similarly, the 2 unripe watermelons in your sample must be chosen from the 9 unripe watermelons.

C(_____, _____) x C(_____, _____) = _____________

# of ways to choose 5 ripe watermelons from 6 ripe available	# of ways to choose 2 unripe watermelons from 9 unripe available

Note: $C(6, 5)$ without $C(9, 2)$ is incorrect since this only accounts for choosing 5 things in your sample. When dealing with counting problems, you must choose everything that is in your sample.

Recall the problem situation: At a fruit stand there are 15 watermelons, of which 6 are ripe. A sample of 7 watermelons is selected at random.

d. How many samples of 7 would have exactly 4 or exactly 5 ripe watermelons?

The word "or" here breaks up this problem into two separate cases, which can be combined using the union rule.

n(exactly 4 $\cup$ exactly 5 ripe) = n(exactly 4 ripe) + n(exactly 5 ripe) – n(exactly 4 ripe $\cap$ exactly 5 ripe)

Note: Since you cannot have exactly 4 AND exactly 5 ripe watermelons in the same sample, then n(exactly 4 ripe $\cap$ exactly 5 ripe) = 0. Therefore,

n(exactly 4 ripe $\cup$ exactly 5 ripe) = C(____, 4)C(____, ____) + $C(6, 5)C(9, 2)$ = _______________

(choose 4 ripe; choose 3 unripe; 5 ripe, 2 unripe)

e. How many samples contain at least 1 ripe watermelon?

"At least 1" means "1 or more" ripe watermelons in the sample. This problem can be handled as in **d.**

n(exactly 1 ripe $\cup$ exactly 2 ripe $\cup$ exactly 3 ripe $\cup$ … $\cup$ exactly 6 ripe)
= n(exactly 1 ripe) + n(exactly 2 ripe) + n(exactly 3 ripe) + … + n(exactly 6 ripe)

Again there is no intersection to subtract off because all cases are mutually exclusive.
Notice that even though the size of your sample is 7, you cannot have exactly 7 ripe watermelons since there are only 6 ripe watermelons at the fruit stand.

This method requires a large computation. An alternative method is to consider the complement of what you are counting.

What is the complement of "at least 1 ripe"? To see this, look at all the possible scenarios for a sample.

Possible Samples of 7

0 ripe, 7 unripe
___ ripe, ___ unripe
___ ripe, ___ unripe
___ ripe, 4 unripe "at least 1 ripe"
4 ripe, ___ unripe
___ ripe, ___ unripe
6 ripe, 1 unripe

Looking at the above scenarios, the complement of "at least 1 ripe" is 0 ripe (and 7 unripe).

Take the total number of samples and subtract off the number of samples in the complement. In other words, subtract the number of samples that you do *not* want.

n(at least 1 ripe) = *n*(total samples) – *n*(exactly 0 ripe)

= *C*(15, _____) – *C*(_____, 0)*C*(_____, _____) = _______________

choose any 7 from all watermelons — choose 0 ripe — choose 7 unripe

Note: Since *C*(6, 0) =1, it is not necessary to write this piece above, although it helps in organization.

4. A school held auditions for the school play. Eleven (11) students tried out for an acting role and 12 students tried out to be an extra. The play requires 2 acting roles (1 lead and 1 supporting) and 5 extras. In how many ways can the positions in the play be determined?

______ x ______ x *C*(_____, _____) = ____________

choose lead — choose supporting — choose 5 extras

Note: The two blanks for choosing the lead and supporting role above can be replaced by one blank and calculated as *P*(11, 2) since 2 of the 11 students are being chosen and "arranged" into their specific roles. *C*(11, 2) is incorrect since it would give the number of ways to choose 2 students from 11, but it does not specify which student gets which role.

5. A jar contains 10 coins, 12 marbles, and 6 erasers. If 8 items are selected at random from the jar, how many samples would contain exactly 2 coins or exactly 6 marbles?

n(exactly 2 coins $\cup$ exactly 6 marbles)
= *n*(exactly 2 coins) + *n*(exactly 6 marbles) – *n*(exactly 2 coins $\cap$ exactly 6 marbles)

a. If there are exactly 2 coins in a sample, how many items in the sample are not coins? _______

b. *n*(exactly 2 coins) = ________________________
choose 2 coins and 6 that are not coins

c. If there are exactly 6 marbles in a sample, how many items in the sample are not marbles? _______

d. *n*(exactly 6 marbles) = ________________________
choose 6 marbles and 2 that are not marbles

e. Is it possible to have exactly 2 coins and exactly 6 marbles in a sample at the same time? ________

f. *n*(exactly 2 coins $\cap$ exactly 6 marbles) = ______________________
choose 2 coins AND 6 marbles

g. *n*(exactly 2 coins $\cup$ exactly 6 marbles) =

______________________ + ______________________ – ______________________

= ______________________

