NO SOLN OR PARAMETRIC

2.3 Systems of Linear Equations: Underdetermined and Overdetermined Systems

Example:

$$\begin{aligned} x+2y+z&=-2\\ -2x-3y-z&=1\\ 2x+4y+2z&=-4 \end{aligned}$$

x y z = #

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & -2 \\ -2 & -3 & -1 & 1 \\ 2 & 4 & 2 & -4 \end{array}\right] \xrightarrow{\text{rref}}$$

Now check is this in RREF?

Are the rows of all zeros is below the non-zero rows?

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Is the first non-zero entry in any row is a 1?

Do the leading 1's go down in a diagonal?

If a column has a leading 1 then does the rest of the column have zeros?

A column containing a leading 1 is called a *unit column* and the variable associated with the column is a *basic variable.*

x y ~~z~~ t $= \#$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Solve for the basic variables & non-basic are parameters

$x - t = 4 \Rightarrow x = t + 4$

$y + t = -3 \Rightarrow y = -t - 3$

$(x, y, z) = (t+4, -t-3, t)$ t is any $\mathbb{R}$

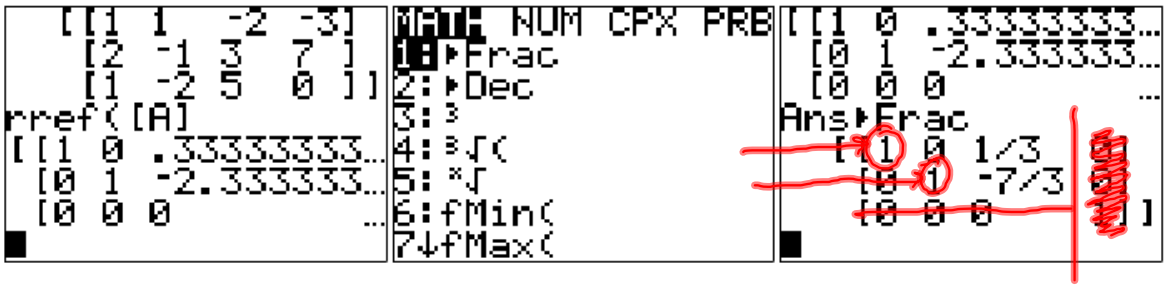
Part. solns: $t=0 \Rightarrow (0+4, -0-3, 0) = (4, -3, 0)$

$t=1 \Rightarrow (1+4, -1-3, 1) = (5, -4, 1)$

Is $(5, -3, 0)$ a part soln? NO

Example:

$$\begin{aligned} x+y-2z&=-3\\ 2x-y+3z&=7\\ x-2y+5z&=0 \end{aligned} \longleftrightarrow \left[\begin{array}{ccc|c} 1 & 1 & -2 & -3\\ 2 & -1 & 3 & 7\\ 1 & -2 & 5 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 1/3 & 0\\ 0 & 1 & -7/3 & 0\\ 0 & 0 & 0 & 1 \end{array}\right]$$



RREF form?

Are the rows of all zeros below the non-zero rows?
Is the first non-zero entry in any row is a 1?
Do the leading 1's go down in a diagonal?
If a column has a leading 1 then is the rest of the column is zero?

$$\left[\begin{array}{ccc|c} 1 & 0 & 1/3 & 0\\ 0 & 1 & -7/3 & 0\\ 0 & 0 & 0 & 1 \end{array}\right] \qquad \begin{aligned} x+\tfrac{1}{3}t&=0\\ y-\tfrac{7}{3}t&=0\\ 0&=1 \end{aligned}$$

$z = t$ = #

FALSE

$\Rightarrow$ NO SOLN

Number of Solutions Theorem

Case 1: If the number of equations is greater than or equal to the number of variables in a linear system then one of the following is true:

- The system has no solution
- The system has exactly one solution
- The system has infinitely many solutions

Case 2: If there are fewer equations than variables then the system has no solution or infinitely many solutions.

Example: Solve the following system (Case 2)

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 2 \\ x_1 + x_2 + 2x_3 &= 1 \end{aligned}$$

$\leftrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 4 & 2 \\ 1 & 1 & 2 & 1 \end{array}\right]$ ref $\rightarrow$

$x_3 = t$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \end{array}\right]$

$x_1 = 0$

$x_2 + 2t = 1 \rightarrow x_2 = -2t + 1$

$(x_1, x_2, x_3) = (0, -2t+1, t)$ t any $\mathbb{R}$

Example: Solve the following system (Case 1):

$$\begin{aligned} 4x &+ 6y = 8 \\ 3x &- 2y = -7 \\ x &+ 3y = 5 \\ 2x &+ 6y = 10 \end{aligned}$$

4x3 ref → error

skip

A company is buying three kinds of vehicles. Carts hold 3 people and cost \$9,000, vans hold 8 people can cost \$27,000 and minivans hold 7 people and cost \$27,000. The company needs to seat 48 people and has \$162,000 to purchase vehicles. How many of each type of vehicle can be purchased?

x = # of carts
y = # of vans
z = # of mini

$3x + 8y + 7z = 48$ (total seats)

$9000x + 27000y + 27000z = 162{,}000$ (total \$)

$$\left[\begin{array}{ccc|c} 3 & 8 & 7 & 48 \\ 9 & 27 & 27 & 162 \end{array}\right] \xrightarrow{\text{ref}} \left[\begin{array}{ccc|c} 1 & 0 & -3 & 0 \\ 0 & 1 & 2 & 6 \end{array}\right]$$

$x - 3t = 0 \to x = 3t$

$y + 2t = 6 \to y = 6 - 2t$

$(x, y, z) = (3t, 6-2t, t)$ t is the # of minivans

$t = 0 \Rightarrow (0, 6, 0)$ Buy 0 carts, 6 vans, and 0 minivans

$t = 1 \Rightarrow (3, 4, 1)$ Buy 3 " 4 " " 1 "

$t = 2 \Rightarrow (6, 2, 2)$ 6 2 " 2 "

$t = 3 \Rightarrow (9, 0, 3)$ 9 0 " 3 "