

### 2.3 Systems of Linear Equations: Underdetermined and Overdetermined Systems

Example:

$$\begin{aligned} x + 2y + z &= -2 \\ -2x - 3y - z &= 1 \\ 2x + 4y + 2z &= -4 \end{aligned}$$

Now check is this in RREF?

Are the rows of all zeros is below the non-zero rows?

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Is the first non-zero entry in any row is a 1?  
Do the leading 1's go down in a diagonal?

If a column has a leading 1 then does the rest of the column have zeros?

A column containing a leading 1 is called a *unit column* and the variable associated with the column is a *basic variable*.

$$\left[ \begin{array}{ccc|c} 1 & 0 & -1 & 4 \\ 0 & 1 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

Example:

$$\begin{aligned} x + y - 2z &= -3 \\ 2x - y + 3z &= 7 \\ x - 2y + 5z &= 0 \end{aligned}$$

[[1 1 -2 -3]	NUM	CPX	PRB	[[1 0 .33333333...
[2 -1 3 7]	Frac			[0 1 -2.333333...
[1 -2 5 0]]	Dec			[0 0 0
rref([A])	3:			Ans>Frac
[[1 0 .33333333...	4:↵			[[1 0 1/3 0]
[0 1 -2.333333...	5:↵			[0 1 -7/3 0]
[0 0 0	6:fMin<			[0 0 0 1]]
	7:fMax<			

RREF form?

Are the rows of all zeros below the non-zero rows?

Is the first non-zero entry in any row is a 1?

Do the leading 1's go down in a diagonal?

If a column has a leading 1 then is the rest of the column is zero?

### Number of Solutions Theorem

Case 1: If the number of equations is greater than or equal to the number of variables in a linear system then one of the following is true:

- The system has no solution
- The system has exactly one solution
- The system has infinitely many solutions

Case 2: If there are fewer equations than variables then the system has no solution or infinitely many solutions.

Example: Solve the following system (Case 2)

$$x_1 + 2x_2 + 4x_3 = 2$$

$$x_1 + x_2 + 2x_3 = 1$$

Example: Solve the following system (Case 1):

$$4x + 6y = 8$$

$$3x - 2y = -7$$

$$x + 3y = 5$$

$$2x + 6y = 10$$

A company is buying three kinds of vehicles. Carts hold 3 people and cost \$9,000, vans hold 8 people and cost \$27,000 and minivans hold 7 people and cost \$27,000. The company needs to seat 48 people and has \$162,000 to purchase vehicles. How many of each type of vehicle can be purchased?