

Markov Chains

A Markov chain or process describes an experiment consisting of a finite number of stages.

- The outcomes and associated probabilities at each stage depend only on the outcomes of the preceding stage.
- The outcome at any stage of the experiment in a Markov chain is called the state of the experiment.

Example

Every Mother's Day, a certain individual sends her mother either roses or carnations. If she sends roses one year, 30% of the time she will send roses again the next year. If she sends carnations one year, 80% of the time she will send roses the next year.

- a) Is this a Markov chain?
- b) If there is a 40% chance of giving roses this year, what is the probability that she sends roses in next year? In two years?

A transition matrix T is a matrix such that:

- The matrix is square
- All entries are nonnegative.
- The entries in each column sum to 1.
- The entries represent conditional probabilities

The initial state is stored as matrix X_0 .

The matrix X_i represents the distribution after i stages.

$$X_n = T^n X_0$$

- c) What is the probability that she will send roses in 10 years?

- d) Given she sent carnations this year, what is the probability that she will give carnations again 3 years from now?

Example

A survey indicates that people in a certain area take their summer vacations either at the beach, at the lake, or in the mountains. The survey finds that among people who have gone to the beach, 20% go to the beach the next summer, 30% go to the lake, and 50% go to the mountains. Among those who have gone to the lake, 20% go to the beach the next summer, 20% go to the lake, and 60% go to the mountains. Finally among those who have gone to the mountains, 50% go to the beach next summer and 50% go to the lake.

- a) Find the transition matrix for this Markov process.
- b) There was an oil spill last summer and no one went to the beach. Instead, half went to the mountains and half to the lake. What is the probability that a people will go to each of these locations two summers after the spill?
- c) What will be the long term distribution of vacation locations?

Example

What is the long term distribution for flowers on Mother's Day?

Example

A company offers three different cars to its executives each year. Those who have a brand A car ask for a brand A car again 30% of the time, they ask for a brand B car 30% of the time and a brand C car 40% of the time. Those who are driving a brand B car ask for a brand A car 50% of the time and a brand C car 50% of the time. Those who are driving a brand C car ask for a brand C car all of the time.

- a) Find the transition matrix for this Markov process.
- b) What is the long term distribution of cars?

A transition matrix T is a regular Markov chain if the sequence T, T^2, T^3, \dots approaches a steady state matrix with all positive entries.

Example

Classify the following matrices as regular, absorbing, or not a transition matrix.

- a) $\begin{bmatrix} 0.9 & 0.2 \\ 0.1 & 0.8 \end{bmatrix}$
- b) $\begin{bmatrix} 0.7 & 1 \\ 0.3 & 0 \end{bmatrix}$
- c) $\begin{bmatrix} 1 & 0.5 \\ 0 & 0.5 \end{bmatrix}$