Last Name: ___________ First Name: _____________ Section 141 - _____ UIN _____________

Basics of Probability

An experiment consists of randomly selecting one letter from the word COMPUTER.

1. Find the set of all possible outcomes, the **sample space**, for this experiment. Separate each outcome with a comma.

$S = \{$________________________________$\}$

2. **Events** are *subsets* of the sample space *S*. Find the following events using the sample space generated in **1.**

a. Let *V* be the event that a vowel is selected. $V = \{$____________$\}$

b. Let *W* be the event that a consonant is selected. $W = \{$__________________$\}$

c. Let *X* be the event that a "T" is selected. $X = \{$________$\}$

d. Let *Y* be the event that a "U" is selected. $Y = \{$________$\}$

e. How many *total* events are associated with this experiment? _________
Hint: How many *subsets* does the set *S* have?

3. Notice that some of the events above contained exactly one outcome and some contained more than one outcome. A **simple event** is an event that consists of exactly one sample point or outcome.

a. How many total *simple* events are there for this experiment? __________

List all of these simple events. ____

b. Which of the events in **2.** are simple events? _______________

c. Can events *X* and *Y* occur at the same time? ________

d. What is the event $X \cap Y$? ________

e. Are *X* and *Y* mutually exclusive events? ________

All simple events are mutually exclusive.

A **uniform sample space** is a sample space in which *all* outcomes are equally likely.

4. **a.** Fill in the remaining information in the table below regarding the experiment in **1**.

Outcome	C	O	M	P	U	T	E	R
Frequency	1	1	________	________	________	________	________	________
Probability (Relative Frequency)	$\frac{1}{8}$	________	________	________	________	________	________	________

b. Is this a uniform sample space? __________ Why or why not?
____

c. What is the sum of the numbers in the second row? ____________

5. a. Fill in the remaining information in the table below regarding the experiment of choosing one letter from the word TELEVISION.

Outcome	T	E	L	V	I	S	O	N
Frequency	1	2	________	________	________	________	________	________
Probability (Relative Frequency)	$\frac{1}{10}$	________	________	________	________	________	________	________

b. Is this a uniform sample space? __________ Why or why not?
____

c. What is the sum of the numbers in the second row? ____________

The tables above are examples of **probability distributions.**

A probability distribution table has the following properties:

- **The outcomes in the table make up the entire sample space.**
- **The outcomes in the table cannot occur at the same time (the corresponding simple events are all mutually exclusive).**
- **The sum of all the probabilities is equal to 1.**

Once you know the probability of each outcome, to find the probabilities of larger events, add up the probabilities of each outcome in the event.

6. Recall *X* is the event that a "T" is selected and *Y* is the event that a "U" is selected from our original experiment in **1**.

a. What is the *event* $X \cup Y$? ____________

b. $P(X \cup Y) =$ _________

c. $P(X) + P(Y) =$ ________ + ________ = ________

d. Does $P(X \cup Y) = P(X) + P(Y)$? ________

e. Since *X* and *Y* are mutually exclusive events, $P(X \cap Y) =$ ________

If *E* and *F* are mutually exclusive events, $P(E \cup F) = P(E) + P(F)$.

An experiment consists of randomly selecting a letter from the word MATHEMATICS.

7. **a.** How many letters are there in this word? __________
b. Are the letters M and C equally likely to be selected? Why or why not? __________________
____

8. Find the set of all possible outcomes in a *uniform* sample space *S* for this experiment.
S = {_________________________________}

Hint: In order to create a *uniform* sample space for this experiment, write every letter (repeated or not) as a separate outcome.

11. What is the probability of randomly choosing

a. the letter H? _________

b. the letter M or the letter C? _________

c. the letter A? _________

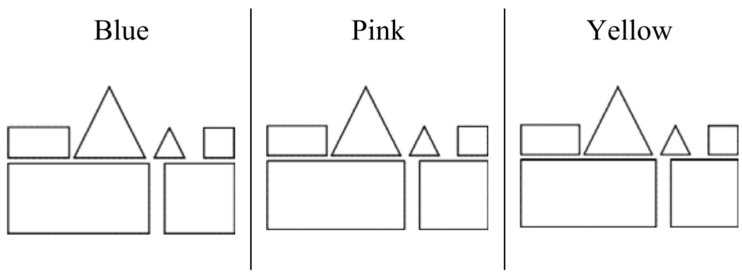
d. a letter that is not A? _________

Note: Your answers from **c.** and **d.** above should sum to one, since these events are complements of one another and they combine to form the entire sample space.

For any event *E*, $P(E)+P(E^C)=1$**, and thus** $P(E^C)=1-P(E)$ **and** $P(E)=1-P(E^C)$**.**

12. In a class, the probability of choosing a female student at random is 0.281. What is the probability of choosing a male student at random from this class? __________

Consider the set of shapes shown below. For each color there are 6 different shapes, a large and a small triangle, a large and a small square and a large and a small rectangle.



13. Suppose an experiment consists of selecting one of these shapes at random.

a. How many possible outcomes are there? ________

b. How many simple events are associated with this experiment? ________

c. How many total events are possible from this experiment? _________

14. Suppose *B* is the event that a blue shape is selected.
a. How many outcomes are in the event *B*? ________

b. What is the probability of *B*? $P(B) =$ ________

15. If *M* is the event that a small shape is selected,
a. How many outcomes are in the event *M*? ________

b. What is $P(M)$? ________

16. Consider the event $B \cap M$.
a. What kind of shape has been selected in order for this event to occur?
The shape is both ____________ **and** ______________.

b. How many outcomes are in the event $B \cap M$? ________

c. What is $P(B \cap M)$? ________

17. Consider the event $B \cup M$.

a. What kind of shape has been selected in order for this event to occur?
The shape is ___________ **or** ____________ **or** both.

b. How many outcomes are in $B \cup M$? ________

c. What is $P(B \cup M)$? ________

d. What is $P(B) + P(M)$? ________+ _________ = _________

e. Does $P(B \cup M) = P(B) + P(M)$? _________
Why or why not? *Hint:* Refer to your answer to **4c**.

____

____

18. If E and F are any two events in a sample space S, how can the terms $P(E)$, $P(F)$, $P(E \cup F)$ and $P(E \cap F)$ be related in a formula? *Hint:* This is called the UNION RULE for probability. Use I for the intersection symbol and U for the union symbol.

____

Note: This formula is consistent with the one boxed at the end of **Part I** since $P(E \cap F) = 0$ for mutually exclusive events.

Part IV

A class has 150 students and the maximum grade possible in this class is 100. Eleven students had a grade of 90 or more. Forty-one students had grades of 80 or more. Fifty-seven students had a grade that was greater than or equal to 60 but less than 70. Ten students had grades less than 60.

If x represents a student's grade, organize this information in the probability distribution table started for you below. Refer to the properties of a probability distribution table in **Part I**.
Hint: Represent the grade ranges by using inequalities. Use <= for ≤ and use >= for ≥. Let x be the student's grade.

Grade, x	________	________	________	$80 \le x < 90$	$90 \le x \le 100$
Frequency	________	________	________	________	11
Probability (Relative Frequency)	________	________	________	________	$\frac{11}{150}$

19. What is the probability a student had a grade less than 80? _____________

20. What is the probability a student had a grade of at least 70? _____________

21. What is the probability a student had a grade of at least 70 but less than 90? _____________