Exam 1 Review

1. A company is buying three kinds of vehicles. Carts hold 3 people and cost $9,000, vans hold 8 people can cost $27,000 and minivans hold 7 people and cost $27,000. The company needs to seat 48 people and has $162,000 to purchase vehicles. How many of each type of vehicle can be purchased?

\[ x = \# \text{ of carts} \]
\[ y = \# \text{ of vans} \]
\[ z = \# \text{ of minivans} \]

\[ 3x + 8y + 7z = 48 \]
\[ 9000x + 27000y + 27000z = 162000 \]

\[
\begin{bmatrix}
3 & 8 & 7 | 48 \\
9 & 27 & 27 | 162
\end{bmatrix}
\rightarrow
\begin{bmatrix}
0 & -3 & 0 | 0 \\
1 & 2 & 6 | 0
\end{bmatrix}
\]

\[ y - 3t = 0 \]
\[ y + 2t = 6 \]

\[ (x, y, z) = (3t, 6 - 2t, t) \]

\[ t = 0 \rightarrow \text{Buy 0 carts, 6 vans, and 0 minivans} \]
\[ t = 1 \rightarrow \text{Buy 3 carts, 4 vans, and 1 minivan} \]
\[ t = 2 \rightarrow \text{Buy 6 carts, 2 vans, and 2 minivans} \]
\[ t = 3 \rightarrow \text{Buy 9 carts, 0 vans, and 3 minivans} \]
2. Four network consultants, Alan, Maria and Steven, each received a year-end bonus of $10,000 which they invested in a 401K retirement plan. Each employee is allowed to place their investments in three funds – an equity index fund (I), a growth fund (II) and a global equity fund (III). The allocations of the investments for each person in each fund is shown in matrix $A$ (in dollars) for each employee. The returns of the funds after 1 year is shown in matrix $B$. What does the product matrix $AB$ represent?

$$A = \begin{bmatrix}
6000 & 3000 & 3000 \\
2000 & 5000 & 3000 \\
2000 & 3000 & 5000
\end{bmatrix}$$

$$B = \begin{bmatrix}
0.18 \\
0.24 \\
0.12
\end{bmatrix}$$

$$AB = \begin{bmatrix}
\]$$

The product $AB$ is the total money each person earns.
3. Solve the following matrix equation for $X$: $2X + D = BX$.

$$
2X - BX = -D
$$

$$
(aI - B)X = -D
$$

$$
(aI - B)^T(aI - B)X = (aI - B)^T(-D)
$$

$$
X = (aI - B)^{-1}(-D)
$$
4. A cordless drill company has monthly fixed costs of $92,500. If each month 12,000 cordless drills are produced and sold for $130 each, then there is a profit of $795,500.

(a) Find the linear cost function.

(b) Each month how much revenue is generated at the break-even point?

\[ \begin{align*}
\text{Profit} &= \text{Revenue} - \text{Cost} \\
R - C &= R(x = 12000) - C(x = 12000) \\
795,500 &= 130 \cdot 12,000 - (m \cdot 12,000 + 92,500) \\
m &= \frac{56}{12} \\
C(x) &= 56x + 92,500 \quad \text{where } x \text{ is the number of drills and } C \text{ is the cost in dollars} \\
R &= C \quad \text{at the B.E point} \\
130x &= 56x + 92,500 \\
x &= 12.50 \\
R &= 130(12.50) = \$162,500 \\
\text{At the Break-Even point, 12.50 drills are sold. The revenue and the cost are } \$162,500.\]
5. An economy consists of three sectors: crafting (C), fishing (F), and gathering (G). The input-output matrix for this economy is given by Matrix $A$.

(a) What does the entry $a_{ij}$ represent?
(b) Find the value of goods consumed in the internal process of production to satisfy an external demand for 560 units of crafted products, 880 units of fishing, and 260 units of gathered goods. Round to two decimal places.

\[
A = \begin{bmatrix}
    1 & 0.1 & 0.1 \\
    0.2 & 0.3 & 0.3 \\
    0.2 & 0.3 & 0.2
\end{bmatrix}
\]

You need 2 units of fishing to make 1 unit of crafting.

\[
X = (I - A)^{-1} D
\]

\[
D = \begin{pmatrix}
    560 \\
    880 \\
    260
\end{pmatrix}
\]

\[
X = \begin{pmatrix}
    2400 \\
    1800 \\
    1800
\end{pmatrix}
\]

Internal demand = \[
\begin{pmatrix}
    2400 \\
    1800 \\
    1800
\end{pmatrix}
- \begin{pmatrix}
    560 \\
    880 \\
    260
\end{pmatrix} = \begin{pmatrix}
    1440 \\
    1720 \\
    1540
\end{pmatrix}
\]

1440 units of crafting, 1720 units of fishing, and 1540 units of gathering are consumed internally in this economy.
6. Six people were asked their annual income (in thousands of dollars) and the average number of hours they spend watching television each week. The data collected is shown in the following table.

<table>
<thead>
<tr>
<th></th>
<th>Income (x)</th>
<th>10</th>
<th>27</th>
<th>38</th>
<th>55.5</th>
<th>70</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hours watching TV (y)</td>
<td>50</td>
<td>33</td>
<td>28</td>
<td>20</td>
<td>12</td>
<td>6</td>
</tr>
</tbody>
</table>

(a) Find the equation of the least-squares line for this data. If needed, round the coefficients to 4 decimal places.

(b) Using the least-squares equation, estimate the average number of hours of televisions watched per week by a person with an annual income of $45,800

(c) Using the least-squares equation, estimate the annual income (to the nearest dollar) of a person who watches an average of 10 hours of television per week.

(a) Enter x into L1 and y into L2. The STAT -> CALC -> LinReg(ax+b)L1,L2,Y1. Note that the Y1 is found by pressing VARS then arrow to Y-VARS then Function and then choose Y1. This will paste the regression equation into Y1.

\[ y = -0.4723x + 48.4871 \]

(b) To use the un-rounded equation, turn on your stat plots and check that Y1 has the regression equation. Then do ZOOM and scroll down for ZoomStat. The regression equation and scatter plot will be displayed. Then go to CALC and choose value. Enter x - 45.8 to find 26.86 hours of television.

(c) Graph the line y=10 and use CALC and intersect to find x = 81.490674.... Multiply by 1000 to get the salary in dollars, $81,491