**The Poisson Distribution**

How many cars arrive at a toll booth in an hour?

How many items are used from an inventory in a week?

How many red blood cells are in a cc of blood?

How many fish are caught in a lake per day?

These are all events that we can try to find the probability of occurring, but not the probability that they don't occur. To study these we will use the Poisson distribution.

**When the Poisson Probability Applies**

The Poisson probability law will apply if there is a number \( \lambda \) such that within a small fractional unit of measurement, such as time or space.

1. Probability of one count \( \approx \lambda \) (size of the small unit).

2. Probability of two or more counts per some small size of the unit is \( \approx 0 \).

3. The number of occurrences of an event in any one interval of time or space is independent of the number in any other disjoint interval of time or space.

If the Poisson distribution applies, then the probability of \( x \) occurrences per unit measure is approximately

\[
P_x(\lambda) = \frac{\lambda^x}{x!} e^{-\lambda} \quad \text{with a mean of } \mu = \lambda \text{ and } \sigma = \sqrt{\lambda}
\]

Example: Suppose on average there are 16 emergency patients on the 8A to 4P shift of a certain hospital. What is the probability that during any one hour of the shift that

a) zero patients arrive?

b) at most one patient?

c) more than one patient?

d) draw the histogram for \( x = 0 \) to 6
Example: On average 90 hamburgers are sold during the lunch hour at a fast food restaurant. What is the probability that during a certain minute of the lunch hour that
a) zero hamburgers will be sold?
b) at most two hamburgers will be sold?
c) more than 3 hamburgers will be sold?

The binomial distribution can be approximated by the Poisson distribution if $p$, the probability of success in a single trial, is small [generally less than 0.1]. In that case, using $\lambda = np$ will be a good approximation.

Example: The probability of theft on a subway is given as 0.001. What is the probability that 10 of the next 5000 passengers will be robbed? Use both the binomial and Poisson distributions.