WEEK 1 REVIEW – Lines and Linear Models

SLOPE

A VERTICAL line has NO SLOPE. All other lines have

\[
\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Example
Find the slope of the line passing through the points \((-2, 4)\) and \((0, -4)\)

Answer
Let one pair of points be \((x_1, y_1)\) and the other \((x_2, y_2)\). Then

If we assigned our points the other way we would have...
EQUATIONS OF LINES

The formula for the slope of a line can be rearranged to give us the equation for a line.

\[ m = \frac{y - y_1}{x - x_1} \rightarrow y - y_1 = m(x - x_1) \]

This is called the POINT-SLOPE form of a line. If you know a point, \((x_1, y_1)\) that lies on the line and you know the slope, \(m\), of the line, then you can find the equation of the line.

Example
What is the equation of the line passing through the points \((-2, 4)\) and \((0, -4)\)?

Answer
\( m = -4 \) (previous example) Let \((x_1, y_1) = (-2, 4)\)

Let \((x_1, y_1) = (0, -4)\)

When we simplify our point-slope form we are writing the line in the slope-intercept form,

\[ y = mx + b \]

Again, \(m\) is the slope and now \(b\) is the \(y\)-intercept.
The $y$-intercept is the place where the line crosses the $y$-axis. The $x$-intercept is the place where the line crosses the $x$-axis.

Example
Graph the line $y = -4x - 4$ and find the intercepts.

Answer

$Ax + By = C$ is the GENERAL FORM of a line.

Example
Graph the line $3x - 4y = 12$ on paper and on the calculator.

Answer
Two lines are parallel if they have the same slope and different $y$-intercepts, $m_1 = m_2$ and $b_1 \neq b_2$.

Two lines are perpendicular if the product of their slopes is -1,
$$m_1 \cdot m_2 = -1 \text{ or } m_1 = \frac{-1}{m_2}.$$ 

**Example**
Given the line $L_1$ is $y = 2x + 4$,
(a) find a line parallel to $L_1$ that passes through the point $(4, 4)$
(b) find a line perpendicular to $L_1$ that passes through the point $(4, 4)$

**Answer**
APPLICATIONS

Example
In the 1990’s for wages less than the maximum taxable wage base, Social Security contributions by employees are 6.2% of the employee's wages.

a) Find a linear model that expresses the relationship between wages and Social Security contributions for employees earning less than the maximum ($106,800 in 2010).

b) Graph this equation and find the social security contribution for an employee earning $35,000 in wages in a year.

Answer
LINEAR BUSINESS MODELS

Depreciation: the value, $V$, of an item decreases linearly with time. The item has an initial value and then the value decreases by the same amount each time period.

Cost: in a linear cost model the TOTAL cost to make $x$ items is $C(x) = cx + F$. $F$ represents the fixed costs. These are the costs you have even if you make no items. $c$ is the cost to make each unit, called the variable cost.
**Revenue:** in a linear revenue model the revenue from selling $x$ items is $R(x) = sx$. $s$ is the sale price of a single item.

**Profit:** the difference between the money in (revenue) and the money spent (costs) is the profit. $P(x) = R(x) - C(x)$
Supply: in a linear supply model the number of items, $x$, that a company will supply at a price $p$ is given by $S(x) = p = m_s x + b_s$.

Demand: in a linear demand model the number of items, $x$, that consumers will purchase at a price $p$ is given by $D(x) = p = m_d x + b_d$. 
DEPRECIATION

Example
A car is purchased for $18,000 and is kept for 7 years. At the end of 7 years the car is sold for $4000. Find an equation that models the decrease in the value of the car over time. What is the car worth after 3 years?

Answer
Example
Suppose a company manufactures baseball caps. In a day they can produce 100 caps for a total cost of $600. If no caps are produced their costs are $200 per day. The caps sell for $8 each. Find the cost, revenue and profit equations.

Answer
**SUPPLY AND DEMAND**

*Example*
A baker is willing to supply 16 jumbo cinnamon rolls to a café at a price of $1.70 each. If she is offered $1.50 for each roll, she will supply 4 fewer roles to the café. At the café, customers will purchase no cinnamon rolls if the cost is $7.20 each. However, if the price of a cinnamon roll is $0.80, the café can sell 40 of these rolls.

Find the supply and demand equations for jumbo cinnamon rolls.
THE INTERSECTION OF TWO LINES

Find where the lines $10x + 4y = 20$ and $3x - y = 12$ intersect.
**Break-even Point:** This is where the cost to produce $x$ items is the same as the revenue brought in from selling these $x$ items. This occurs when $R(x) = C(x)$.

*Example*
Find and interpret the break-even point for making and selling baseball caps.
Equilibrium Point: This is the price $p$ that the consumer and producer are willing to pay/accept for $x$ items. This occurs when $S(x) = D(x)$

Example
Find and interpret the equilibrium point for the supply and demand for jumbo cinnamon rolls.
**QUADRATICS**

A quadratic is a polynomial of order 2:

\[ y = ax^2 + bx + c, \quad a \neq 0. \]

Every quadratic function can also be written in standard form:

\[ y = a(x - h)^2 + k \]

where \( h = -\frac{b}{2a} \) and \( k = c - \frac{b^2}{4a} \)

Note the vertex is a min \( a > 0 \)

Note the vertex is a max \( a < 0 \)

Concave up

Concave down

The x-intercepts can be found using the quadratic formula:

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

when \( b^2 - 4ac \geq 0 \)
Graph the following quadratics and find the intercepts and vertices.

\[ y = x^2 + x - 12 \quad y = 4x^2 + 16x + 2 \quad y = -x^2 + 4x - 5 \]
Example: What is the revenue from selling espressos if the demand equation for selling espressos is \( p = -0.25x + 5 \)? Graph the revenue equation and interpret the result.