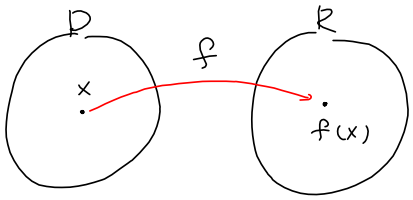
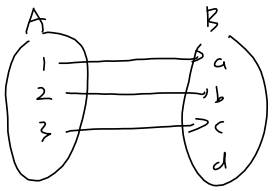
Chapter 1. Functions

Section 1.1 and A.8

Definition. A **function** is a rule that assigns to each element x in the domain exactly one element in the range.

The **domain** is the set of all values of the *independent variable* (typically x) that produce real values for the *dependent variable* (typically y). The **range** is the set of all possible y values as x varies throughout the domain. If the correspondence is a function we use the notation $f(x)$ read "f of x". Ordered pair of the form (x, y) can be written as $(x, f(x))$.





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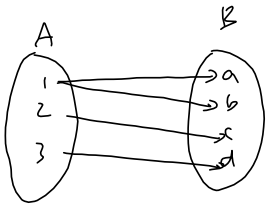

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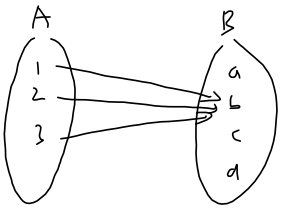

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Ex1) Find the domain of the following functions.

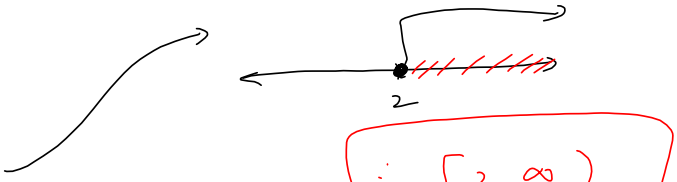
1. $f(x) = \sqrt{2x-4}$ ≥ 0

$\Rightarrow 2x - 4 \geq 0$

$+4 \quad +4$

$\Rightarrow \frac{2x}{2} \geq \frac{4}{2}$

$\Rightarrow x \geq 2$



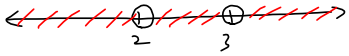
$\therefore [2, \infty)$

2. $g(x) = \dfrac{x-1}{x^2-5x+6} = \dfrac{x-1}{(x-2)(x-3)}$

$A = -5 : -2, -3$

$M = 6$

Domain: $\mathbb{R}$ except $x = 2, 3$



$(-\infty, 2) \cup (2, 3) \cup (3, \infty)$

3. $h(x) = \dfrac{\sqrt{5x-6}}{\sqrt[3]{3x-5}}$

Intersection

$3x - 5 \neq 0$

$+5 \quad +5$

$\Rightarrow \frac{3x}{3} \neq \frac{5}{3}$

$\Rightarrow x \neq \frac{5}{3}$

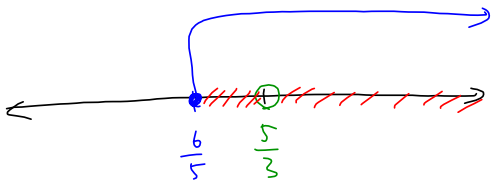
And

$5x - 6 \geq 0$

$+6 \quad +6$

$\Rightarrow \frac{5x}{5} \geq \frac{6}{5}$

$\Rightarrow x \geq \frac{6}{5}$



$\therefore \left[\frac{6}{5}, \frac{5}{3}\right) \cup \left(\frac{5}{3}, \infty\right)$

In Sum,

The domain of most algebraic functions is the set of all real numbers EXCEPT:

1. Any real numbers that cause the denominator of a function to be 0. $\frac{x}{x-4}$
2. Any real numbers that cause us to take the square root (or any radical with an even index) of a negative number.

and more...

Definition. Functions with domains divided into two or more parts with a different rule applied to each part are called **piecewise-defined functions**.

The absolute value function, $|x|$, is such an example.

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

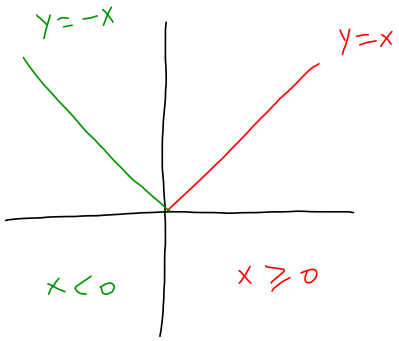
$|2| = 2$

$|-4| = -(-4) = 4$

Graph of a functions

The **graph** of a function f consists of all points (x, y) such that x is in the domain of f and $y = f(x)$.

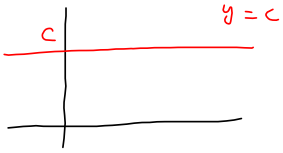
Ex2) Graph the function $f(x) = |x|$.



Catalog of Basic Functions

1. Constant function

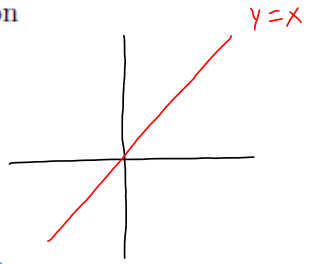
$f(x) = c, \quad c$: real constant



Domain: $(-\infty, \infty)$
Range: $\{c\}$

2. Identity function

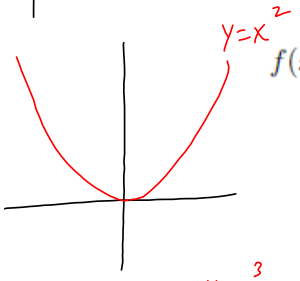
$f(x) = x$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

3. Square function

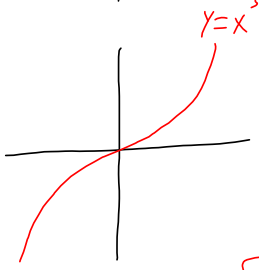
$f(x) = x^2$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$

4. Cubing function

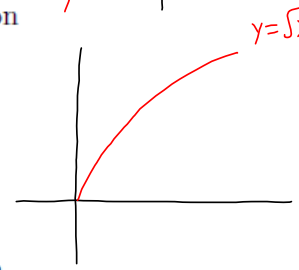
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$

5. Square root function

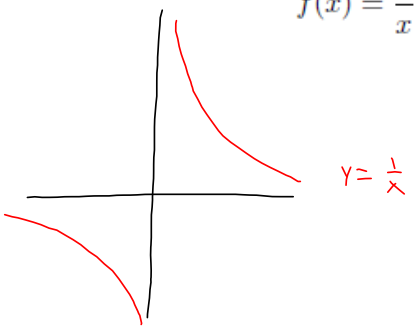
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$

6. Reciprocal function

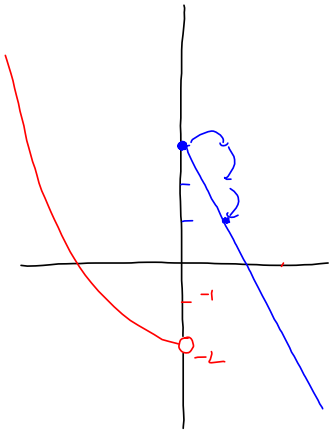
$f(x) = \frac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$

Ex3) Graph the following piecewise defined function:

$$f(x) = \begin{cases} x^2 - 2 & \text{if } x < 0 \\ -2x + 3 & \text{if} x \geq 0 \end{cases}$$



Vertical line test

A graph in the xy-plane represents a function of x, if and only if, every vertical line intersects the graph in at most one place.

Finding domain and range from graphs

- To find the domain of f, vertical line scanning the x-axis.
- To find the range of f, horizontal line scanning the y-axis.