

Domain of Radical

$$\sqrt{x}, \quad x \geq 0$$

Rationalizing Denominators

: Removing the radical in the denominator.

1. The radical is a single term.

Multiply both the numerator and the denominator of the expression by something which will produce a perfect root in the denominator.

$$\text{Ex) } \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}} \cdot 1 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}, \quad \frac{1}{\sqrt[3]{5}} = \frac{1}{\sqrt[3]{5}^{\frac{1}{3}}} \cdot 1 = \frac{1}{\sqrt[3]{5}^{\frac{1}{3}}} \cdot \frac{5^{\frac{2}{3}}}{5^{\frac{2}{3}}} = \frac{5^{\frac{2}{3}}}{5^1} = \frac{\sqrt[3]{5^2}}{5}$$

Note that the denominator in the last fraction contains no radical.

Ex13) Rationalizing Denominators.

$$\text{a) } \frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \cdot 1 = \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt[3]{3}}{3} = \sqrt{3}$$

$$\text{b) } \frac{1}{\sqrt[3]{x^2}} = \frac{1}{x^{\frac{2}{3}}} \cdot 1 = \frac{1}{x^{\frac{2}{3}}} \cdot \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} = \frac{x^{\frac{1}{3}}}{x^{\frac{2}{3}}} = \frac{\sqrt[3]{x}}{x}$$

$$\text{c) } \sqrt[7]{\frac{1}{a^2}} = \frac{\sqrt[7]{1}}{\sqrt[7]{a^2}} = \frac{1}{a^{\frac{2}{7}}} \cdot 1 = \frac{1}{a^{\frac{2}{7}}} \cdot \frac{a^{\frac{5}{7}}}{a^{\frac{5}{7}}} = \frac{a^{\frac{5}{7}}}{a^{\frac{2}{7}}} = \frac{\sqrt[7]{a^5}}{a^{\frac{2}{7}}} = \frac{\sqrt[7]{a^5}}{a}$$

$$\text{d) } \frac{1}{\sqrt[4]{x^4 y^8 z^3}}, \quad (z > 0) = \frac{1}{(x^4 y^8 z^3)^{\frac{1}{4}}} = \frac{1}{(x^4)^{\frac{1}{4}} (y^8)^{\frac{1}{4}} (z^3)^{\frac{1}{4}}} \\ = \frac{1}{|x| \cdot y^2 \cdot z^{\frac{3}{4}}} \cdot 1 = \frac{1}{|x| y^2 z^{\frac{3}{4}}} \cdot \frac{z^{\frac{1}{4}}}{z^{\frac{1}{4}}} = \frac{\sqrt[4]{z}}{|x| \cdot y^2 \cdot z}$$

2. The Denominator is a Sum of Terms.

If the denominator has a sum of radical form such as

$$a + b\sqrt{m},$$

multiply both the numerator and denominator by the conjugate of the denominator.

Note. Conjugate of $a + b\sqrt{m}$ is $a - b\sqrt{m}$.

$$\text{Ex) Conjugate of } 2 + \sqrt{3} = 2 - \sqrt{3} \\ \text{ " " } \sqrt{2} + \sqrt{5} = \sqrt{2} - \sqrt{5}$$

Ex) Rationalizing Denominator using Conjugate

$$\frac{3}{1 - \sqrt{2}} = \frac{3}{1 - \sqrt{2}} \cdot \frac{1 + \sqrt{2}}{1 + \sqrt{2}} = \frac{3 + 3\sqrt{2}}{1 - 2} = \frac{3 + 3\sqrt{2}}{-1} = -3 - 3\sqrt{2}$$

Ex14) Rationalize the denominator.

$$\text{a) } \frac{4}{\sqrt{5} + \sqrt{6}} \cdot 1 = \frac{4}{\sqrt{5} + \sqrt{6}} \cdot \frac{\sqrt{5} - \sqrt{6}}{\sqrt{5} - \sqrt{6}} = \frac{4(\sqrt{5} - \sqrt{6})}{5 - 6} = \frac{4\sqrt{5} - 4\sqrt{6}}{-1} \\ = -4\sqrt{5} + 4\sqrt{6}$$

$$\text{b) } \frac{\sqrt{5}}{4 - \sqrt{11}} \cdot 1 = \frac{\sqrt{5}}{4 - \sqrt{11}} \cdot \frac{4 + \sqrt{11}}{4 + \sqrt{11}} = \frac{\sqrt{5}(4 + \sqrt{11})}{16 - 11} \\ = \frac{4\sqrt{5} + \sqrt{55}}{5}$$

3. Combining Radical Expressions

Radical expressions can only be combined by addition or subtraction if there are like terms—terms with the same index and same radicand (the expression inside the radical).

$$\text{Ex) } \underline{\sqrt{2}} + \underline{\underline{4\sqrt{7}}} + \underline{2\sqrt{2}} - \underline{\underline{3\sqrt{7}}}$$

$$= (1+2)\sqrt{2} + (4-3)\sqrt{7}$$

$$= 3\sqrt{2} + \sqrt{7} \quad (\text{Note that the two remaining terms are not alike and, hence, cannot be simplified more}).$$

Ex15) Simplify the expression.

a) $\sqrt{50} - \sqrt{32} + \sqrt{2}$

$$= \sqrt{5 \cdot 2} - \sqrt{4 \cdot 2} + \sqrt{2} = \underbrace{5\sqrt{2} - 4\sqrt{2} + \sqrt{2}}_{\text{like term}}$$

$$= (5-4+1)\sqrt{2}$$

$$= 2\sqrt{2}$$

b) $7\sqrt[3]{80x} - 2\sqrt[3]{270x} + 4\sqrt[3]{10}$

$$= 7 \cdot \sqrt[3]{2^3 \cdot 10 \cdot x} - 2 \cdot \sqrt[3]{3^3 \cdot 10 \cdot x} + 4 \cdot \sqrt[3]{10}$$

$$= 7 \cdot 2 \cdot \sqrt[3]{10x} - 2 \cdot 3 \cdot \sqrt[3]{10x} + 4 \cdot \sqrt[3]{10}$$

$$= \underbrace{14 \cdot \sqrt[3]{10x} - 6 \cdot \sqrt[3]{10x}}_{\text{like term}} + 4 \cdot \sqrt[3]{10} = (14-6) \cdot \sqrt[3]{10x} + 4 \cdot \sqrt[3]{10}$$

$$= 8 \cdot \sqrt[3]{10x} + 4 \cdot \sqrt[3]{10}$$

c) $\frac{2}{\sqrt{6}} + \frac{3\sqrt{3}}{\sqrt{2}} - \frac{2}{\sqrt{3}}$

$$= \frac{2}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} + \frac{3\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} - \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \underbrace{\frac{2\sqrt{6}}{6} + \frac{3\sqrt{6}}{2} - \frac{2\sqrt{3}}{3}}_{\text{like term}}$$

$$= \left(\frac{2}{6} + \frac{3}{2}\right)\sqrt{6} - \frac{2}{3}\sqrt{3} = \left(\frac{2}{6} + \frac{9}{6}\right)\sqrt{6} - \frac{2}{3}\sqrt{3}$$

$$= \frac{11}{6}\sqrt{6} - \frac{2}{3}\sqrt{3}$$

