Chapter 1. Basic Algebraic Concepts.

Chapter 1A. Real Numbers.

Number Systems

- **Natural Number**
  or Counting Number

$$\mathbb{N} = \{1, 2, 3, 4, 5, \cdots\}$$

- **Whole Number**

$$\mathbb{W} = \{0, 1, 2, 3, 4, 5, \cdots\}$$

$\mathbb{N} \subset \mathbb{W}$

- **Integers**

$$\mathbb{Z} = \{\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots\}$$

- **Rational Number** A rational number is a number that can be expressed as a **fraction** with an integer numerator and a non-zero integer denominator.

$$\mathbb{Q} = \left\{ \frac{m}{n} \middle| m, n \text{ integers}, n \neq 0 \right\}$$

ex) $4 = \frac{4}{1}$

$\frac{1}{4} = 0.25$

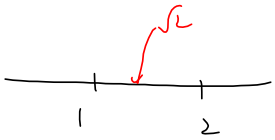
$\frac{1}{3} = 0.333\cdots = 0.\overline{3}$

Note. Rational numbers have repeating or terminating decimal expansion.

$\frac{1}{3} = 0.333\ldots$

- **Irrational Number** If a number cannot be written as a fraction of two integers, it is called irrational number. But still it is real number.

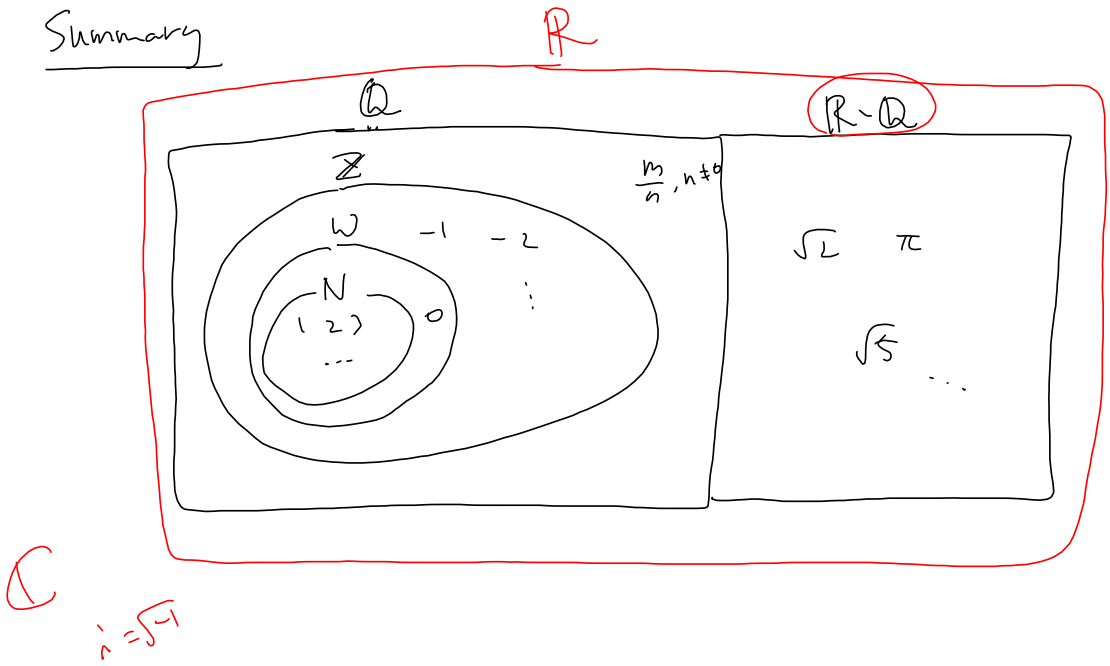
ex) $\sqrt{2} = 1.41\ldots$



$\pi = 3.14\ldots$

Real #.

Summary



Ex1) For the following, list of numbers, classify each according to what type(s) of number(s) it is.

$=\sqrt{9^2}=9$

	$-7,$	$0.46,$	$0.78\overline{94},$	$\pi^2,$	$\sqrt{81},$	$\sqrt[4]{8},$	$1,$	$\frac{12}{7}$
Real	✓	✓	✓	✓	✓	✓	✓	✓
Rational	✓	✓	✓		✓		✓	✓
Irrational				✓		✓		
Integer	✓				✓		✓	
Natural					✓		✓	

Properties of Real Numbers

Closed.

For any two real numbers a and b, the sum $a + b$ and the product $a \cdot b$ are uniquely defined real numbers. And these operations satisfy the properties of real numbers.

1. **Commutative:** $a + b = b + a, \quad a \cdot b = b \cdot a$

   ex) $2+4 = 4+2$, $2 \cdot 3 = 3 \cdot 2$

2. **Associative:** $a + (b + c) = (a + b) + c, \quad a(bc) = (ab)c$

   ex) $2+(4+5) = (2+4)+5$, $2(3 \cdot 4) = (2 \cdot 3) \cdot 4$

3. **Identity:** $a + 0 = 0 + a = a, \quad a \cdot 1 = 1 \cdot a = a$

   Additive identity Multiplicitive identity

   ex) $4 + 0 = 4$, $9 \cdot 1 = 9$

4. **Inverse:**

   - Additive inverse is opposite of a.

     $$a + (-a) = (-a) + a = 0.$$

     ex) $5 + (-5) = 0$

   - Multiplicative inverse is reciprocal of a.

     $$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1.$$

     ex) $\frac{7}{1} \cdot \left(\frac{1}{7}\right) = 1$, $\frac{2}{3} \cdot \left(\frac{3}{2}\right) = 1$

5. **Distributive:** $a(b + c) = ab + ac, \quad (a + b)c = ac + bc$

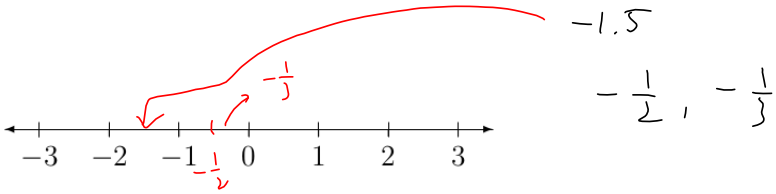
   ex) $2(4+5) = 2 \cdot 4 + 2 \cdot 5 = 8 + 10 = 18$.

   $(-4)(2+3) = (-4) \cdot 2 + (-4) \cdot 3 = -8 - 12 = -20$.

   $(2-3) \cdot 4 = 2 \cdot 4 + (-3) \cdot 4 = 8 - 12 = -4$.

Number Lines and Absolute Value.

Each point on the number line corresponds to exactly one real number.



The **absolute value** of a number x, denoted by $|x|$, refers to the distance from that number to the origin.

$|-2| = 2$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$|x-6| = \begin{cases} x-6 & \text{if } x-6 \geq 0 \\ -(x-6) & \text{if } x-6 < 0 \end{cases}$$

or $x \geq 6$

or $x < 6$

ex) $|4| = 4$

$|-4| = -(-4) = 4$

$|3-6| = |-3| = 3$

$= |6-3|$

In general, $|a-b|$ or $|b-a|$ gives us the distance between the numbers a and b on the number line.

ex) $(-2)\cdot(-1)\cdot 3 = +6$ ex) $(-2x)\cdot x\cdot(6)$

$(-2)\cdot(-1)(-3) = -6$ $= -12x^2$.

Ex2) What is the distance between the numbers 2 and -24?

$|2-(-24)| = |2+24| = |26| = 26$

or $|-24-2| = |-26| = -(-26) = 26$

$|a-b|$

Ex3) Find the distance between the numbers -1 and $\frac{3}{2}$.

$\left|-1-\frac{3}{2}\right| = \left|-\frac{2}{2}-\frac{3}{2}\right| = \left|\frac{-2-3}{2}\right| = \left|\frac{-5}{2}\right| = \frac{5}{2}$

or $\left|\frac{3}{2}+(+1)\right| = \left|\frac{3}{2}+1\right| = \left|\frac{3}{2}+\frac{2}{2}\right| = \left|\frac{3+2}{2}\right| = \left|\frac{5}{2}\right| = \frac{5}{2}$ ✓

Ex4) Find the distance between the numbers $-\frac{3}{4}$ and $-\frac{2}{7}$.

$\left|-\frac{3}{4}+\left(+\frac{2}{7}\right)\right| = \left|-\frac{3}{4}\cdot 1+\frac{2}{7}\cdot 1\right| = \left|-\frac{3}{4}\cdot\frac{7}{7}+\frac{2}{7}\cdot\frac{4}{4}\right|$

$= \left|-\frac{21}{28}+\frac{8}{28}\right| = \left|\frac{-21+8}{28}\right|$

or $\left|-\frac{2}{7}-\left(-\frac{3}{4}\right)\right| = \cdots = \frac{13}{28}$.

$= \left|\frac{-13}{28}\right| = \frac{13}{28}$.

Ex5) Evaluate

$$\left|-24+(-24)\right|+75+(-49)-\left|74\right|$$