Ex2) What is the distance between the numbers 2 and -24?

Ex3) Find the distance between the numbers -1 and $\frac{3}{2}$.

Ex4) Find the distance between the numbers $-\frac{3}{4}$ and $-\frac{2}{7}$.

Ex5) Evaluate

$$\left|-24+(-24)\right|+75+(-49)-\left|74\right|$$

$$= \left|-24-24\right| + 75 - 49 - 74$$

$$= \left|-48\right| + 75 - 49 - 74$$

$$= 48 + 75 - 49 - 74$$

$$= (48 - 49) + (75 - 74)$$

$$= -1 + 1 = 0$$

Ex6) Evaluate

$$\frac{-2-\left|-1-\left|-\frac{8}{5}\right|\right|}{\frac{3}{4}}$$

$= -\frac{5}{5} - \frac{8}{5} = \frac{-5-8}{5} = -\frac{13}{5}$

$= \frac{8}{5}$

$$= \frac{-2-\left|-1-\frac{8}{5}\right|}{\frac{3}{4}} = \frac{-2-\frac{13}{5}}{\frac{3}{4}} = \frac{-2\cdot 1-\frac{13}{5}}{\frac{3}{4}}$$

$$= \frac{-\frac{2}{1}\cdot\frac{5}{5}-\frac{13}{5}}{\frac{3}{4}} = \frac{\frac{-10}{5}-\frac{13}{5}}{\frac{3}{4}} = \frac{\frac{-10-13}{5}}{\frac{3}{4}} = \frac{-\frac{23}{5}}{\frac{3}{4}}$$

$$= \cancel{\frac{-23}{5}\div\frac{3}{4}} = \frac{-23}{5}\cdot\frac{4}{3} = -\frac{92}{15}$$

Ex7) Evaluate

$$\left(2\frac{2}{5}\right)\left|2\frac{2}{3}-5\frac{1}{8}\right|$$

$2\frac{2}{5} = \frac{10}{5}+\frac{2}{5} = \frac{12}{5}$

$2\frac{2}{3} = \frac{6}{3}+\frac{2}{3} = \frac{8}{3}$

$5\frac{1}{8} = \frac{40}{8}+\frac{1}{8} = \frac{41}{8}$

$$= \frac{12}{5}\cdot\left|\frac{8}{3}-\frac{41}{8}\right|$$

$$= \frac{12}{5}\cdot\left|\frac{8}{3}\cdot 1-\frac{41}{8}\cdot 1\right|$$

$$= \frac{12}{5}\cdot\left|\frac{8}{3}\cdot\frac{8}{8}-\frac{41}{8}\cdot\frac{3}{3}\right|$$

$$= \frac{12}{5}\cdot\left|\frac{64}{24}-\frac{123}{24}\right|$$

$$= \frac{12}{5}\cdot\left|\frac{64-123}{24}\right|$$

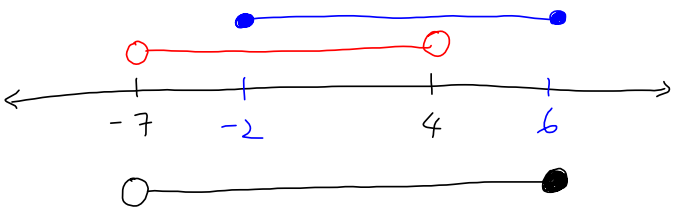
$$= \frac{12}{5}\cdot\left|\frac{-59}{24}\right| = \frac{\cancel{12}^{1}}{5}\cdot\frac{59}{\cancel{24}_{2}}$$

$$= \frac{59}{10}.$$

Ex8) Graph the following sets and write the solution in interval notation.

a)

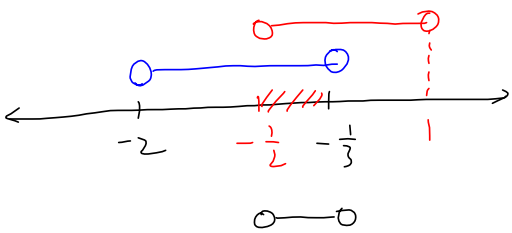
$$(-7, 4) \cup [-2, 6]$$



$\therefore\ -7 < x \leq 6$ or $(-7, 6]$.

b)

$$\left(-2, -\frac{1}{3}\right) \cap \left(-\frac{1}{2}, 1\right)$$



$\therefore\ -\frac{1}{2} < x < -\frac{1}{3}$ or $\left(-\frac{1}{2}, -\frac{1}{3}\right)$

Chapter 1B Exponents and Radicals

Properties of Exponents

1) If n is a positive integer,

$$x^n = \underbrace{x \cdot x \cdots x}_{n}$$

ex) $7^4 = 7 \cdot 7 \cdot 7 \cdot 7$

$(-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8$

$(-2)^4 = (-2)(-2)(-2)(-2) = 16$

$(-3)^2 = 9$

$-(3)^2 = -9$

2) If $n = 0$, then we define

$$x^0 = 1$$

ex) $1000\,000\,000^0 = 1$

3) We define x $(x \neq 0)$ to a negative power,

reciprocal.

$$x^{-1} = \frac{1}{x}$$

$$x^{-2} = \frac{1}{x^2}$$

$$x^{-n} = \frac{1}{x^n}$$

$$2^{-2} = \frac{1}{2^2}$$

$$\left(\frac{2}{3}\right)^{-2} = \left(\frac{3}{2}\right)^2 = \frac{3^2}{2^2} = \frac{9}{4}$$

4)

ex) $3^2 \cdot 3^3 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 3^5 \leftarrow 3^{2+3}$

$$a^m \cdot a^n = a^{m+n}$$

5)

$$\frac{2^4}{2^2} = 2^{4-2} = 2^2$$

or $\frac{2^{4-2}}{\cancel{2^2}\,1} = \frac{2^2}{1} = 2^2$

$$\frac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$$

6)

ex) $(7^2)^4 = 7^8$

$$(a^m)^n = a^{mn}$$

7)

$(2 \cdot 3)^4 = 2^4 \cdot 3^4$

$$(ab)^n = a^n b^n$$

8)

$$\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2}$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$$

Ex9) Simplify the expression

a) $(-x)^4(-2xy)^3 = (-1)^4 \cdot x^4 \cdot (-2)^3 \cdot x^3 y^3$ or $(+x)^4$

$= x^4 \cdot (-8)\, x^3 \cdot y^3$

$= -8 \cdot x^{4+3} \cdot y^3$

$= -8x^7 \cdot y^3$

b) $\left(\frac{x^{-2}y^3}{3y^{-1}}\right)^{-3} = \frac{x^6 \cdot y^{-9}}{3^{-3} \cdot y^3} = \frac{x^6 \cdot 3^3}{y^9 \cdot y^3} = \frac{27x^6}{y^{9+3}} = \frac{27x^6}{y^{12}}$.

c) $8^4 \cdot 2^{-3} = (2^3)^4 \cdot 2^{-3}$

$= 2^{12} \cdot 2^{-3} = 2^{12-3} = 2^9$

$8 = 2^3$

2 4

2 2

d) $15^{-2} \cdot 3^2 = (3 \cdot 5)^{-2} \cdot 3^2$

$= 3^{-2} \cdot 5^{-2} \cdot 3^2$

$= 3^{-2+2} \cdot 5^{-2}$

$= 3^0 \cdot 5^{-2} = 1 \cdot 5^{-2} = 5^{-2} = \frac{1}{5^2}$.

$15 = 3 \cdot 5$

3 5

e) $5^3 \cdot 4^{-2} \cdot 10^2 \cdot 7^3 = 5^3 \cdot (2^2)^{-2} \cdot (2 \cdot 5)^2 \cdot 7^3$

$= 5^3 \cdot 2^{-4} \cdot 2^2 \cdot 5^2 \cdot 7^3$

$= 2^{-4+2} \cdot 5^{3+2} \cdot 7^3$

$= 2^{-2} \cdot 5^5 \cdot 7^3 = \dfrac{5^5 \cdot 7^3}{2^2}$

f) $\dfrac{(198)^3 \cdot (700)^{-4}}{5^4 \cdot 6^{-3}} = \dfrac{(2 \cdot 3^2 \cdot 11)^3 \cdot (2^2 \cdot 5^2 \cdot 7)^{-4}}{5^4 \cdot (2 \cdot 3)^{-3}}$

$198 = 2 \cdot 3^2 \cdot 11$

$198 \to 2, 99 \to 3, 33 \to 3, 11$

$700 = 2^2 \cdot 5^2 \cdot 7$

$700 \to 2, 350 \to 2, 175 \to 5, 35 \to 5, 7$

$= \dfrac{2^3 \cdot 3^6 \cdot 11^3 \cdot 2^{-8} \cdot 5^{-8} \cdot 7^{-4}}{5^4 \cdot 2^{-3} \cdot 3^{-3}}$

$= 2^{3+(-8)+(+3)} \cdot 3^{6+(+3)} \cdot 5^{-8-4} \cdot 7^{-4} \cdot 11^3$

$= 2^{-2} \cdot 3^9 \cdot 5^{-12} \cdot 7^{-4} \cdot 11^3$

$= \dfrac{3^9 \cdot 11^3}{2^2 \cdot 5^{12} \cdot 7^4}$

or $2^{21-22} = 2^{-1} = \frac{1}{2}$

g) $\dfrac{\left(\frac{1}{2}\right)^{-22} - 8^7}{4^{12} + 4^{11}} = \dfrac{2^{22} - (2^3)^7}{(2^2)^{12} + (2^2)^{11}} = \dfrac{2^{22} - 2^{21}}{2^{24} + 2^{22}} = \dfrac{\cancel{2^{21}}^{\,1}(2^1 - 1)}{2^{\cancel{22}\,1}(2^2 + 1)}$

$= \dfrac{1 \cdot (2-1)}{2(2^2+1)} = \dfrac{1}{2 \cdot 5} = \dfrac{1}{10}.$

Radicals and Properties of Radicals

Radicals (or roots) are, in effect, the opposite of exponents. In other words, the nth root of a number a is a number b such that

$$b = \sqrt[n]{a} = a^{\frac{1}{n}} \Leftrightarrow \quad b^n = a.$$

The number b is called an nth **root of** a. The number n is refered to as the index of the radical (if no index appears, n is 2).

The principal n^{th} root of a number is the nth root of a which has the same sign as a. For example, both 2 and -2 satisfy $x^2 = 4$, but 2 is the (principal) square root of 4.

$\sqrt[2]{}$

$\sqrt{4} = \sqrt{2^{\cancel{2}}} = 2$ or $(4)^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^{2\cdot\frac{1}{2}} = 2^1 = 2$

$\sqrt[3]{8} = \sqrt[3]{2^{\cancel{3}}} = 2$ or $(8)^{\frac{1}{3}} = (2^3)^{\frac{1}{3}} = 2^{3\cdot\frac{1}{3}} = 2^1 = 2$

$\sqrt[3]{-64} = (-64)^{\frac{1}{3}} = ((-4)^3)^{\frac{1}{3}} = -4.$

Properties of nth Roots

1) $\sqrt[n]{ab} = (ab)^{\frac{1}{n}} = a^{\frac{1}{n}} \cdot b^{\frac{1}{n}} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

2) $= \left(\frac{b}{a}\right)^{\frac{1}{n}} = \frac{b^{\frac{1}{n}}}{a^{\frac{1}{n}}} = \frac{\sqrt[n]{b}}{\sqrt[n]{a}}$

$$\sqrt[n]{\frac{b}{a}} = \frac{\sqrt[n]{b}}{\sqrt[n]{a}}$$

3)

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$(a^{\frac{1}{n}})^{\frac{1}{m}} = a^{\frac{1}{nm}} = \sqrt[nm]{a}.$

4)

$$\sqrt[n]{a^n} = a \quad \text{if } n \text{ is odd}$$

5)

$$\sqrt[n]{a^n} = |a| \quad \text{if } n \text{ is even}$$

6)

$$\sqrt[m]{a^n} = a^{\frac{n}{m}}$$