MATH 150 Pre-Calculus

Fall, 2014, WEEK 10

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Week 10: 6A, 6B, 7A, 7B

Chapter 6. Exponentials and Logarithms Revisited

Chapter 6A. Exponentail and Logarithmic Equations

Exponentail Equations

An **exponential equation** is one in which the variable occurs in the exponent. For example,

$$2^x = 7$$

Guidelines for solving Exponential Equations

1. Isolate the exponential expression on one side of the equation.
2. Take the logarithm of each side, then use the Law of Logarithms to "bring down the exponent".
3. Solve for the variable.

Ex1) Find the solution of the equation $3^{x+2} = 7$

Ex2) Solve $4 + 3^{x+1} = 8$.

Ex3) Solve the equation $\dfrac{10}{1+e^{-x}} = 2$.

Ex4) Solve the equation $3xe^x + x^2e^x = 0$.

Ex5) Solve the equation $e^{2x} - 3e^x + 2 = 0$.

Ex6) Solve the equation $e^{2x} - e^x - 6 = 0$.

Logarithmic Equations

A **logarithmic equation** is one in which a logarithm of the variable occurs. For example,

$$\log_2(x+2) = 5$$

Guidelines for solving Logarithmic Equations

1. Isolate the logarithmic term on one side of the equation: you may first need to combine the logarithmic terms.
2. Write the equation in exponential form (or raise the base to each side of the equation)
3. Solve for the variable.

Ex7) Solve $\log x = 35$ for x

Ex8) Solve $\ln(x - 3) = 5$ for x

Ex9) Solve $6 - \log_5(3x - 2) = 4$ for x

Ex10) Solve the equation $\log_2 3 + \log_2 x = \log_2 5 + \log_2(x - 2)$

Ex11) Solve $\log x + \log(x-1) = \log(4x)$

Ex12) Solve for x: $\log(3x-10) = 2 + \log(x-2)$

Chapter 6B. Applications of Exponentials and Logarithms

Exponential Growth Model

A population that experiences **exponential growth** increases according to the model

$$P(t) = P_0 \cdot e^{rt}$$

where
$P(t)$ =population at time t
$P_0 = P(0)$ =initial size of population
r =relative rate of growth
t =time.

Ex13) If $P(t) = 6 \cdot 5^{2t}$, then $P(t)$ satisfies an exponential growth law. What is P_0. Find a value of t such that $P(t) = 150$.

Ex14) The initial bacterium count in a culutre is 500. A biologist later makes a sample count of bacteria in the culture and finds that the relative rate of growth is 40 % per hour.

a) Find a function that models the number of bacteria after t hours.

b) What is the estimated count after 10 hours.

Ex15) A certain breed of rabbit was introduced onto a small island about 8 years ago. The current rabbit population on the island is estimated to be 4100, with a relative growth rate of 55 % per year.

a) What was the initial size of the rabbit population?

b) Estimate the population 12 years from now.

Ex16) The population of the world in 2000 was 6.1 billion, and the estimated relative growth rate was 1.4 % per year. If the population continues to grow at this rate, when will it reach 122 billion?

Ex17) A culture starts with 10,000 bacteria, and the number doubles every 40 minute, a) Find the number of bacteria after one hour, and b) after how many minutes will there be 50,000 bacteria?

Radioactive Decay

Radioactive substances decay by spontaneously emitting radiation. The rate of decay is directly proportional to the mass of the substance. This is analogous to population growth, except that the mass of radioactive material *decreases*. It can be shown that the mass $m(t)$ remaining at time t is modeled by the function

$$m(t) = m_0 e^{-rt}$$

where r is the rate of decay expressed as a proportion of the mass and m_0 is the initial mass. Physicists express the rate of decay in terms of **half-life**, the time required for half the mass to decay.

Radioactive Decay Model

If m_0 is the initial mass of a radioactive substance with half-life h, then the mass remaining at time t is modeled by the function

$$m(t) = m_0 e^{-rt}$$

where $r = \dfrac{\ln 2}{h}$.

Ex18) The half life of uranium 235 is 7.1×10^8 years. If we start out with 1.5 kilograms of ^{235}U in 2014, how much uranium will be left after 10,000 years?

Ex19) Suppose a radioactive substance satisfies the exponential decay law $A(t) = A(0) \cdot 4^{-t}$, where t is in centuries. What is the half-life of this substance?

Newton's Law of Cooling

If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the object at time t is modeled by the function

$$T(t) = T_s + D_0 \cdot e^{-kt}$$

where k is a positive constant that depends on the type of object.

Ex20) A cup of coffee has a temperature of $200°F$ and is place in a room that has a temperature of $70°F$. After 10 minutes the temperature of the coffee is $150°F$.

a) Find the temperature of the coffee after 15 minutes.

b) When will the coffee have cooled to $100°F$.

Chapter 7. Systems of Equations

Chapter 7A. Systems of Linear Equations

A **system of equations** is a set of equations that involves the same variables. A **solution** of a system is an assignment of values for the variables that makes each equation in the system true.

To **solve** a system means to find all solutions of the system.

Substitution Method

1. **Solve for One Variable.** Choose one equation and solve for one variable in terms of the other variable.

2. **Substitute.** Substitute the expression you found in Step 1 into the other equation to get an equation in one variable, then solve for that variable.

3. **Back-Substitute.** Substitute the value you found in Step 2 back into the expression found in Step 1 to solve for the remaining variable.

Ex21) Find all solutions of the system.

$$2x + y = 1$$

$$3x + 4y = 14$$

Ex22) Solve the system

$$x - 3y = 6$$

$$2x + 5y = 7$$

Ex23) Solve the system

$$2x - 6y = 5$$

$$-x + 3y = 4$$

Elimination Method

1. **Adjust the Coefficients.** Multiply one or more of the equations by appropriate numbers so that the coefficient of one variable in one equation is the negative of its coefficient in the other equation.

2. **Add the Equations.** Add the two equations to eliminate one variable, then solve for the remaining variable.

3. **Back-Substitute.** Substitute the value you found in Step 2 back into one of the original equations, and solve for the remaining variable.

Ex24) Find all solutions of the system

$$3x + 2y = 14$$

$$x - 2y = 2$$

Ex25) Solve the system

$$5x + 2y = 8$$

$$3x + y = 5$$

Ex26) A plane flies a round trip between two cities. The flight from the first city is into a strong headwind and takes 1 hour and 30 minutes. The return flight is with the wind and take 55 minutes. If the cities are 100 miles apart what is the aircraft's speed, and what is the wind's speed. Assume that both the aircraft's and wind's speeds are constant.

Ex27) A salad dressing manufacturer wants to make a new version of a honey mustard dressing by combining two other honey mustard dressings. Dressing number 1 contains 5 % honey and dressing number 2 contains 4 % honey. How many quarts of each of these dressings must the manufacture combine in order to produce 1000 quarts of a 4.75 % honey dressing?

Chapter 7B.

Ex28) Find all solutions of the system.

$$x^2 + y^2 = 100$$

$$3x - y = 10$$

Ex29) Solve the system of equation

$$y = (x-5)^2 + 7$$

$$(x-5)^2 + (y-7)^2 = 6$$

Ex30) Solve the system

$$16x^2 - 64x + y^2 = -39$$

$$x^2 - 4x + y^2 = 21$$

Ex31) Solve the system

$$y^2 - 4x^2 = 4$$
$$x = y^2 - 4$$

Ex32) Solve the system

$$x^2 + 4y^2 = 36$$

$$2y + x + 6 = 0$$