# Chapter 1D. Rational Expression.

## Definition of a Rational Expression

A rational expression is an expression of the form $\frac{p}{q}$, where $p$ and $q$ are polynomials, and $q$ cannot be the zero polynomial.

ex) $\frac{4x-6}{3x+16}$, $\frac{x^3-x^2+1}{x+1}$, $\frac{4}{1}$ : rational expressions

$\frac{\sqrt{x^2+1}}{x+7}$ : Not rational expression.

**Note.** Restrictions occur whenever the denominator is zero.

Ex1) Find a domain.

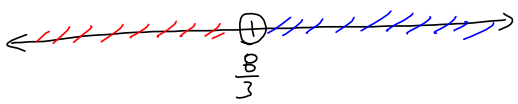
a) $\frac{4x-7}{6x-16}$

since $6x-16=0$
$\quad +16 \quad +16$

$\frac{6x}{6}=\frac{16}{6}$

$x=\frac{16}{6}=\frac{8}{3}$

$\therefore$ The domain is all real #, except $x=\frac{8}{3}$




or $(-\infty, \frac{8}{3}) \cup (\frac{8}{3}, +\infty)$

b) $\dfrac{2}{x-9}$

c) $\dfrac{x+5}{x^2+25}$

All real #.

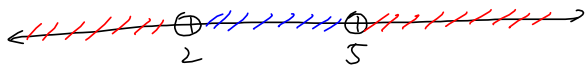
d) $\dfrac{x+1}{x^2-7x+10}$

Since $x^2-7x+10=0 \Rightarrow (x-5)(x-2)=0$

$A=-7$ ; $-5$ $-2$ $\quad x-5=0 \quad x-2=0$

$M=10$ $\quad x=5 \quad x=2$

$\therefore$ The domain is all real # except $x=5, 2$




$(-\infty, 2) \cup (2, 5) \cup (5, +\infty)$

**Simplifying Rational Expressions**

To simplifying a rational expression means to reduce it to lowest terms by cancelling common factors. Therefore, the key to simplifying is to factor the polynomials.

Ex2)

a) $\frac{\cancel{8}^{2}\cancel{x^4}^{1}}{\cancel{12}_{3}\cancel{x^6}_{x^2}} = \frac{2 \cdot 1}{3 \cdot x^2} = \frac{2}{3x^2}$

restriction $x \neq 0$

b) $\frac{x^3+5x^2+6x}{x^2-4} = \frac{x(x^2+5x+6)}{(x+2)(x-2)} = \frac{x\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)}$ restriction $x \neq -2, 2$

$= \frac{x(x+3)}{x-2}$

c) $\frac{4x-x^3}{x^2-x-2} = \frac{x(4-x^2)}{(x+1)(x-2)} = \frac{x\overset{-1}{\cancel{(2-x)}}(2+x)}{(x+1)\cancel{(x-2)}}$ Restriction: $x \neq -1, 2$

$= \frac{-x(x+2)}{x+1} = -\frac{x(x+2)}{x+1}$