MATH 150 PRE-CALCULUS

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JoungDong Kim

Week 5: 3B, 3C

Chapter 3B. Graphs of Equations

Draw the graph $x + y = 6$.

Then every point on the graph satisfies the equation $x + y = 6$.

Note. The graph of any linear equation (First degree polynomial) is a straight line.

Important Concept: The graph of an equation consists of all pairs (x, y) that are solutions to the equation. Every solution to the equation is a point on the graph and every point on the graph (x, y) is a solution to the equation.

Ex0)

a) $(-4, 18)$ is on the graph of $2x + y = 10$? Or is a solution $2x + y = 10$?

b) $(5, 1)$ is on the graph of $2x + y = 10$? Or is a solution $2x + y = 10$?

c) $(4, -2)$ is on the graph of $2x + y = 10$? Or is a solution $2x + y = 10$?

d) $(0, 10)$ is on the graph of $2x + y = 10$? Or is a solution $2x + y = 10$?

To graph an equation by plotting points:

1. Solve the equation for y.

2. Complete a table of values by substituting your choice of values for x into the equation, then solving for y. Use as many points as necessary to determine the shape of the graph.

3. Plot the points and draw a smooth curve throught them.

Ex1) Graph the equation $4x^2 - 2y = 4$.

Ex2) Graph the equation $y = -|x + 1| + 3$.

Ex3) Graph the equation $y = \sqrt{x - 2}$.

Intercepts

The points where the graph of an equation crosses the x-axis is called the x**-intercept**.

The points where the graph of an equation crosses the y-axis is called the y**-intercept**.

Finding the x-intercept.

To find the x-intercept(s) of an equation, substitute 0 for y, and solve for x.

Finding the y-intercept.

To find the y-intercept(s) of an equation, substitute 0 for x, and solve for y.

Ex4) Find the x- and y-intercepts of $y = 2x^2 - 2$ algebraically.

Note. When graphing by plotting points, it is important to include points close to and on either side of the x-intercepts, because the y-values may change sign on either side of a zero.

Ex5) Find the intercepts of the graph of $x^2 + y^2 = 25$.

Symmetry

A shape looks the same on both sides of a dividing line or point.
The dividing line is called the **line of symmetry**.

Definition

A graph is symmetric about the y-axis if and only if for every point (x, y) on the graph, $(-x, y)$ will also be a point on the graph.

Test for Symmetry about the y-axis

To determine algebraically if a graph will be symmetric about the y-axis, **substitute $-x$ for x and simplify**. If the resulting equation is equivalent to the original equation, the graph will be symmetric about the y-axis.

Definition

A graph is symmetric about the x-axis if and only if for every point (x, y) on the graph, $(x, -y)$ will also be a point on the graph.

Test for Symmetry about the x-axis

To determine algebraically if a graph will be symmetric about the x-axis, **substitute $-y$ for y and simplify**. If the resulting equation is equivalent to the original equation, the graph will be symmetric about the x-axis.

Definition

The graph of an equation will have symmetry about the origin if and only if for every point (x, y) on the graph, the point $(-x, -y)$ will also be on the graph.

Test for Symmetry about the origin

Substitute $-x$ for x, and $-y$ for y into the equation. If the resulting equation is equivalent to the original, the graph has symmetry about the orgin.

Ex6) Test the equation $xy^3 - 5x^3y = xy$ for symmetry about the x-axis, y-axis, and origin.

Note. Knowing whether the graph of a particular equation has symmetry about a point or line can assist us if we are graphing by plotting points. If we are using a grapher it provides us information about what our graph should look like so that we can determine whether a particular graph is reasonable for the equation we entered.

Chapter 3C. Linear Equations in Two Variables

Definition Any equation that can be written in the form $Ax + By = C$, where A and B are not both 0, is called a **linear equation**.

Note. The graph of any linear equation is a straight line.

Therefore, we can graph a linear equation by plotting at least two points. The intercepts are good choices because they are usually easy points to find.

Ex7) Graph the linear equation $3x - 2y = 8$.

Ex8) Graph $2y + 6 = 0$ and $2x = 4$.

Slopes of Lines

Definition The **slope** of a line, m, is the ratio of the change in y to the change in x.

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

Definition The slope of the line through points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Ex9) Plot each pair of points and find the slope of the line.

a) $(-3, 5)$ and $(4, -2)$

b) $(5, 7)$ and $(-1, 7)$

c) $(-2, -3)$ and $(5, 6)$

d) $(-2, 5)$ and $(-2, -1)$

Note. For $y = mx + b$, the larger $|m|$, steeper the line.

To graph a line using the $y = mx + b$ form,

1. Since the y-intercept is b, plot the point $(0, b)$ on the y-axis.
2. From the y-intercept, apply the slope, $m = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}}$ to locate a second point on the line
3. Draw the line.

Ex10) Graph $y = -3x - 2$

Horizontal and Vertical Lines

- The slope of a horizontal line is 0.
- The slope of a vertical line is undefined.

Parallel and Perpendicular Lines

- Parallel lines: Equal slopes
- Perpendicular lines: Slopes are negative reciprocals.

Ex11) If the line through $(3, 6)$ and $(2, b)$ is parallel to $3x - 4y = 8$, find the value for b.

Equations of Lines

$$y = mx + b$$

: slope-intercept form of linear equation.

1. **Point-Slope form:**

   The equation of a line with slope m that passes through point (x_1, y_1) is

   $$y - y_1 = m(x - x_1)$$

   Ex12) Write an equation for the line with slope $\frac{1}{2}$ that passes throgh $(2, -1)$.

2. **Point-Point form:**

   The equation of a line passes through points (x_1, y_1) and (x_2, y_2) is

   $$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

   Ex13) Write an equation for the line that passes through $(1, 1)$ and $(2, 3)$

3. **Slope-Intercept form:**

   The equation of a line with slope m and y-intercept b is

   $$y = mx + b$$

   Ex14) Write an equation for the line with slope 5 and y-intercept 2 or $(0, 2)$.

Horizontal lines

The equation of a horizontal line through the point (a, b) is

$$y = b$$

Vertical lines

The equation of a vertical line through the point (a, b) is

$$x = a$$

Ex15) Write an equation for the perpendicular bisector of the line segment connecting $A(-4,1)$ and $B(6,5)$. The perpendicular bisector of a line segment $\overline{AB}$ is the line that is perpendicular to $\overline{AB}$ and cuts $\overline{AB}$ into two equal pieces.