Key

MATH 150, FALL 2014
EXAM II MULTIPLE CHOICE - VERSION A

LAST NAME(print): _____FIRST NAME(print): _____

UIN: _____SECTION NUMBER: _____

DIRECTIONS:

1. This is a 10-question multiple-choice exam; there is no partial credit. Each problem is worth 5 points for a total of 50 points. Mark the correct choice on your ScanTron using a No. 2 pencil. The scantrons will not be returned, therefore *for your own records, also record your choices on your exam!*

2. The use of a calculator and computer is prohibited.

3. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

4. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

5. Your exam grade (sum of both exam parts) will be posted in WebAssign.

6. You may not discuss the contents of the exam with anyone until the exam is returned in class.

THE AGGIE CODE OF HONOR

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Signature: _____

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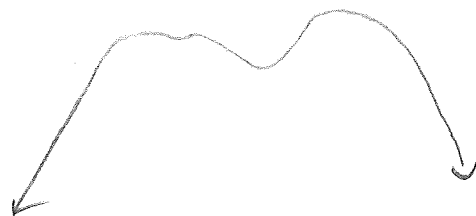
Signature: _____

1. Describe the end behavior of the polynomial $p(x) = -x(x^7 - 4x^5 + 126x^4 + 7x^2 - 1255)$

     (a) $p(x) \to \infty$ as $x \to \infty$ and $p(x) \to \infty$ as $x \to -\infty$

     (b) $p(x) \to -\infty$ as $x \to \infty$ and $p(x) \to -\infty$ as $x \to -\infty$

     (c) $p(x) \to \infty$ as $x \to \infty$ and $p(x) \to -\infty$ as $x \to -\infty$

     (d) $p(x) \to -\infty$ as $x \to \infty$ and $p(x) \to \infty$ as $x \to -\infty$

     (e) $p(x) \to -\infty$ as $x \to \infty$ and $p(x) \to 0$ as $x \to -\infty$

degree : even

leading coeff. : negative

2. What is the equation of the circle that has a diameter with endpoint $(-1, -7)$ and $(5, 2)$?

     (a) $(x - 2)^2 + (y + \frac{5}{2})^2 = \frac{\sqrt{117}}{2}$

     (b) $(x - 2)^2 + (y + \frac{5}{2})^2 = \frac{117}{4}$

     (c) $(x - 2)^2 + (y + \frac{5}{2})^2 = (\frac{117}{4})^2$

     (d) $(x + 2)^2 + (y - \frac{5}{2})^2 = \frac{\sqrt{117}}{2}$

     (e) $(x + 2)^2 + (y - \frac{5}{2})^2 = \frac{117}{4}$

$$\text{Center} = \left(\frac{-1+5}{2}, \frac{-7+2}{2} \right)$$

$$= \left(2, -\frac{5}{2} \right)$$

$$\text{radius} = \sqrt{(-1-2)^2 + (-7 - (-\frac{5}{2}))^2}$$

$$= \sqrt{(-3)^2 + (-\frac{9}{2})^2}$$

$$= \sqrt{9 + \frac{81}{4}} = \sqrt{\frac{117}{4}}$$

$$\therefore (x-2)^2 + (y + \frac{5}{2})^2 = \frac{117}{4}$$

3. Find an equation of the line that is perpendicular to the line $4x + 6y + 5 = 0$ and passes through the y-intercept of the line $2x - 3y = 6$.

     (a) $y = \frac{2}{3}x - 2$

     (b) $y = \frac{3}{2}x - \frac{5}{6}$

     (c) $y = \frac{3}{2}x - 2$

     (d) $y = -\frac{2}{3}x - \frac{5}{6}$

     (e) $y = -\frac{2}{3}x - 2$

-2

$$\text{slope}: -\frac{2}{3} \Rightarrow \text{my slope} = \frac{3}{2}$$

$$y = \frac{3}{2}x - 2$$

4. Allison can build a lego in 2 hours, and James can do the same thing in 6 hours. How long will it take if they build the lego together?

(a) None of these

(b) 1 hour

(c) 1 hour and 20 minutes

(d) 1 and half hour

(e) 1 hour and 40 minutes

$$\left(\frac{1}{2} + \frac{1}{6}\right) t = 1$$

$$\Rightarrow \quad \frac{4}{6} t = 1$$

$$\therefore t = \frac{3}{2} \qquad 1 \text{ har and } 30 \text{ minutes}$$

5. State the domain of $f(x) = \dfrac{\sqrt{2x^2 + 5x - 3}}{x + 3}$ in interval notation.

(a) $(-\infty, -\frac{1}{2}] \cup (3, \infty)$

(b) $(-\infty, -\frac{1}{2}] \cup [3, \infty)$

(c) $(-\infty, -3) \cup (\frac{1}{2}, \infty)$

(d) $(-\infty, -3] \cup [\frac{1}{2}, \infty)$

(e) $(-\infty, -3) \cup [\frac{1}{2}, \infty)$

$$2x^2 + 5x - 3 \geq 0 \qquad \text{and} \quad x \neq -3$$

$$A = 5$$
$$M = -6 \quad : \quad -1 \quad 6$$

$$\Rightarrow 2x^2 - x + 6x - 3 \geq 0$$

$$\Rightarrow x(2x-1) + 3(2x-1) \geq 0$$

$$\Rightarrow (x+3)(2x-1) \geq 0$$

take $x=-1$ | take $x=0$ | take $x=100$
$(-)(-) \geq 0$ | $(+)(-) \geq 0$ | $(+)(+) \geq 0$

$$T \quad -3 \quad F \quad \frac{1}{2} \quad T$$

$$\Rightarrow (-\infty, -3] \cup [\frac{1}{2}, \infty) \quad \text{and} \quad x \neq -3$$

$$\therefore (-\infty, -3) \cup [\frac{1}{2}, \infty)$$

6. Describe the transformations of $y = \frac{5}{4}f(-x) - 3$.

   (a) Vertical stretch by a factor $\frac{5}{4}$, reflect about y-axis, and shift down 3 units.

   (b) Vertical stretch by a factor $\frac{5}{4}$, reflect about x-axis, and shift down 3 units.

   (c) Vertical stretch by a factor $\frac{4}{5}$, reflect about y-axis, and shift down 3 units.

   (d) Vertical stretch by a factor $\frac{4}{5}$, reflect about x-axis, and shift down 3 units.

   (e) Vertical stretch by a factor $\frac{4}{5}$, shift left 1 unit, and shift down 3 units.

7. If $f(x) = \dfrac{2}{x+1}$ and $g(x) = \dfrac{x}{x+1}$, find and simplify $\left(\dfrac{f}{g}\right)(x)$ and state the domain in interval notation.

   (a) $\frac{2x}{(x+1)^2}$, domain is $(-\infty, -1) \cup (-1, \infty)$

   (b) $\frac{2x}{(x+1)^2}$, domain is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

   (c) $\frac{2}{x}$, domain is $(-\infty, 0) \cup (0, \infty)$

   (d) $\frac{2}{x}$, domain is $(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

   (e) None of these

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{2}{x+1} \neq 0}{\frac{x}{x+1} \neq 0} \qquad \text{restric}$$
$$x+1 \neq 0 \qquad : x \neq 0, -1.$$
$$= \frac{2(x+1)}{x(x+1)} = \frac{2}{x}$$

Domain:

$(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

8. If $f(x) = \dfrac{x}{x+1}$ and $g(x) = 2x - 1$, find and simplify $(f \circ g)(x)$ and its domain in interval notation.

   (a) $(f \circ g)(x) = \frac{2x-1}{2x}$, domain is $(-\infty, 0) \cup (0, \infty)$

   (b) $(f \circ g)(x) = \frac{x(2x-1)}{x+1}$, domain is $(-\infty, -1) \cup (-1, \infty)$

   (c) $(f \circ g)(x) = \frac{2x-1}{2x}$, domain is $(-\infty, 0) \cup (0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

   (d) $(f \circ g)(x) = \frac{x(2x-1)}{x+1}$, domain is $(-\infty, -1) \cup (-1, 0) \cup (0, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

   (e) None of these

$$(f \circ g)(x) = f(g(x)) = f(2x-1) = \frac{2x-1}{2x-1+1} = \frac{2x-1}{2x}$$

domain of g : $\mathbb{R}$

domain of $f \circ g$: $(-\infty, 0) \cup (0, \infty)$

9. If a ball is thrown directly upward with a velocity of 40 ft/s, its height (in feet) after t seconds is given by $y = 40t - 16t^2$. What is the maximum height attained by the ball?

(a) 20 ft
(b) 25 ft
(c) 30 ft
(d) 35 ft
(e) 40 ft

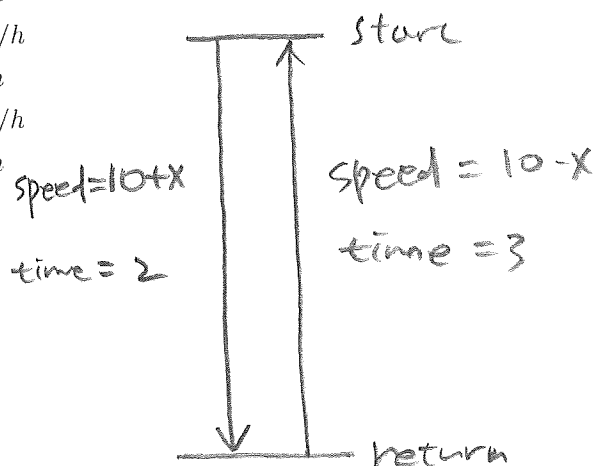$$t = \frac{-b}{2a} = \frac{-40}{2(-16)} = \frac{-40}{-32} = \frac{5}{4}$$

$$y\left(\frac{5}{4}\right) = 40\left(\frac{5}{4}\right) - 16\left(\frac{5}{4}\right)^2$$

$$= 50 - 16 \cdot \frac{25}{16}$$

$$= 50 - 25 = 25$$

10. A motor boat traveled downstream in 2 hours while the return trip upstream took 3 hours. If the speed of boat was 10 $mile/h$, what was the speed of the current?

(a) 2 $mile/h$
(b) 2.5 $mile/h$
(c) 3 $mile/h$
(d) 3.5 $mile/h$
(e) 4 $mile/h$

$speed = 10 + X$

$time = 2$

start

$speed = 10 - X$

$time = 3$

return

$distance = speed \times times$

$(10 + X) \cdot 2 = (10 - X) \cdot 3$

$20 + 2X = 30 - 3X$

$5X = 10$

$\therefore X = 2$

Key

MATH 150, FALL 2014
EXAM II WORK OUT - VERSION A

LAST NAME(print): _____ FIRST NAME(print): _____

UIN: _____ SECTION NUMBER: _____

DIRECTIONS:

1. This is a 9-question work-out exam; Each problem is worth 5 points (or 10 points) for a total of 50 points. Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct.

2. Fully simplify all your answers, and give exact answers unless otherwise stated.

3. Circle your final answer.

4. The use of a calculator and computer is prohibited.

5. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.

6. Your exam grade (sum of both exam parts) will be posted in WebAssign.

7. You may not discuss the contents of the exam with anyone until the exam is returned in class.

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Signature: _____

1. Sketch the graph of the piecewise function.

$$f(x) = \begin{cases} 1 & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ -(x-1)^2 - 1 & \text{if } x > 1 \end{cases}$$
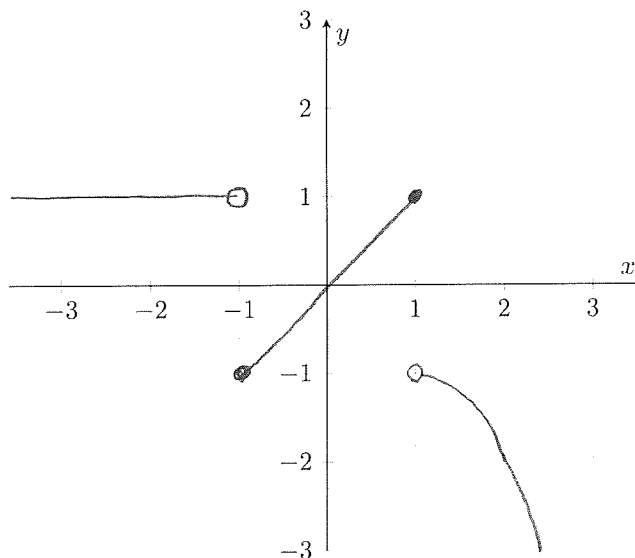
2. Given the circle $\dfrac{2x^2 + 2y^2 + x = 0}{2}$, find the a) center, b) radius, and c) domain in interval notation.

$\Rightarrow x^2 + y^2 + \frac{1}{2}x = 0$

$\Rightarrow \left(x^2 + \frac{1}{2}x\right) + y^2 = 0$

$\Rightarrow \left(x^2 + \frac{1}{2}x + \left(\frac{1}{4}\right)^2\right) + y^2 = \left(\frac{1}{4}\right)^2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 + y^2 = \left(\frac{1}{4}\right)^2$

a) center: $\left(-\frac{1}{4}, 0\right)$

b) radius: $\frac{1}{4}$

c) domain: $\left[-\frac{1}{2}, 0\right]$

2

3. Give the difference quotient, then apply and simplify it for the function $f(x) = \sqrt{x^2 - 4}$.

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{(x+h)^2 - 4} - \sqrt{x^2 - 4}}{h}$$

$$= \frac{\left(\sqrt{(x+h)^2 - 4} - \sqrt{x^2 - 4}\right)}{h} \cdot \frac{\left(\sqrt{(x+h)^2 - 4} + \sqrt{x^2 - 4}\right)}{\left(\sqrt{(x+h)^2 - 4} + \sqrt{x^2 - 4}\right)}$$

$$= \frac{(x+h)^2 - 4 - (x^2 - 4)}{h \cdot \left(\sqrt{(x+h)^2 - 4} + \sqrt{x^2 - 4}\right)}$$

$$= \frac{x^2 + 2xh + h^2 - 4 - x^2 + 4}{h(\quad \text{,,} \quad)}$$

$$= \frac{2xh + h^2}{h(\quad \text{,,} \quad)}$$

$$= \frac{h(2x + h)}{h(\quad \text{,,} \quad)}$$

$$= \frac{2x + h}{\sqrt{(x+h)^2 - 4} + \sqrt{x^2 - 4}}$$

4. (10 points) a) Find the inverse function of $f(x) = 1 + \sqrt{1+x}$, and simplify. b) State the inverse's domain and range in interval notation. c) Graph $f(x)$ and $f^{-1}(x)$ on the same coordinate plane.

$$y = 1 + \sqrt{1+x}$$

$$x = 1 + \sqrt{1+y}$$

$$x - 1 = \sqrt{1+y}$$

$$(x-1)^2 = 1+y$$

$$y = (x-1)^2 - 1$$

$$\therefore f^{-1}(x) = (x-1)^2 - 1$$

domain of f : $1 + x \geq 0 \Rightarrow x \geq -1$
         or $[-1, \infty)$

range of f : $y \geq 1$ or $[1, \infty)$

$\Rightarrow$ domain of f^{-1} : $[1, \infty)$

range of f^{-1} : $[-1, \infty)$

a) inverse function: $\underline{f^{-1}(x) = (x-1)^2 - 1}$

b) domain: $\underline{[1, \infty)}$ range: $\underline{[-1, \infty)}$

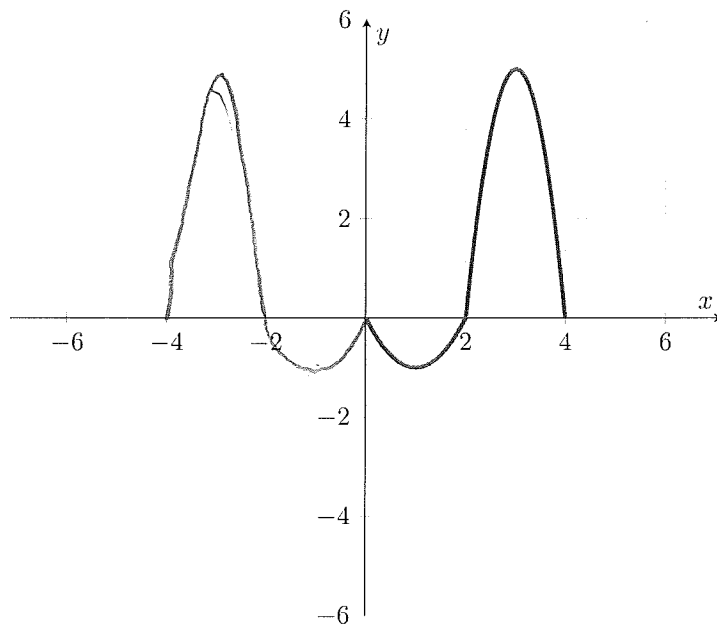c) graph

$y = f^{-1}$

$y = f$
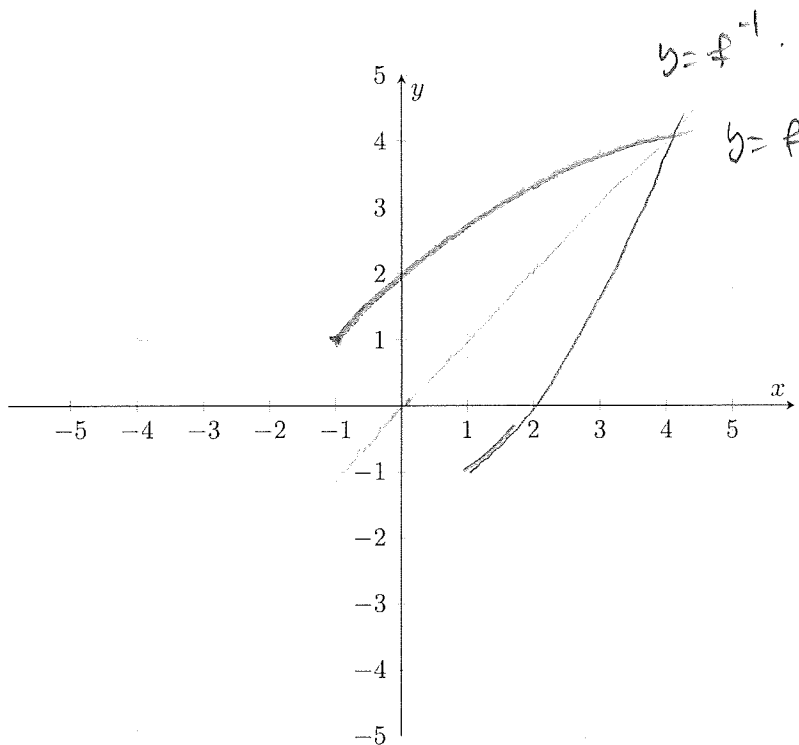
5. What is the a) domain, b) range, c) x-intercepts, d) y-intercepts, e) maximum value, f) minimum value, g) increasing interval, h) decreasing interval, and i) constant interval of the graph below?
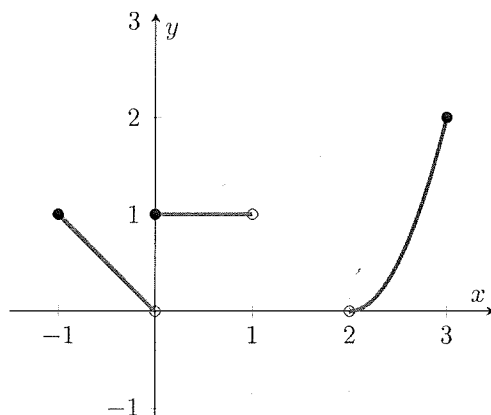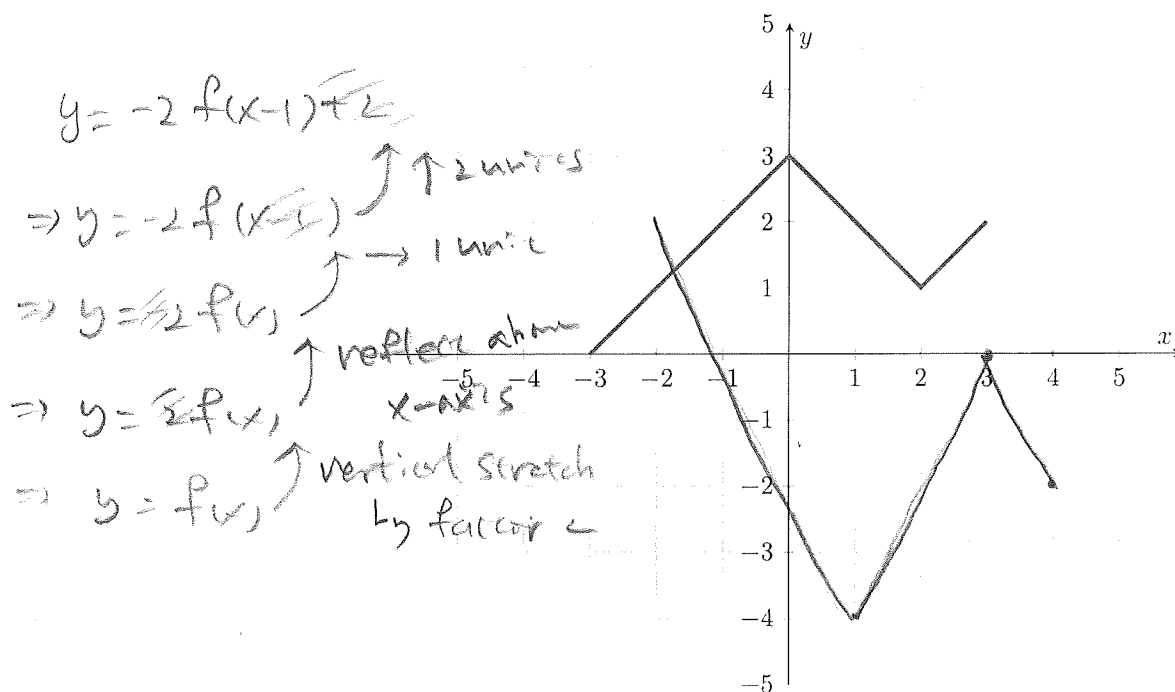

a) domain: $[-1, 1) \cup (2, 3]$

b) range: $(0, 2]$

c) x-intercepts: none

d) y-intercepts: $(0, 1)$

e) max: 2

f) min: none

g) increasing interval (interval notation): $(2, 3]$

h) decreasing interval (interval notation): $[-1, 0)$

i) constant interval (interval notation): $[0, 1)$

6. The graph of a function defined for $x \geq 0$ is given. Complete the graph for $x < 0$ to make an even function.

7. The graph of f is given. Sketch the graph $y = -2f(x-1) + 2$ on the same coordinate plane.

$y = -2f(x-1) + 2$

$\Rightarrow y = -2f(x-1)$ ↑↑ 2 units

$\Rightarrow y = -2f(x)$ ↑→ 1 unit

$\Rightarrow y = 2f(x)$ ↑ reflect about x-axis

$\Rightarrow y = f(x)$ ↑ vertical stretch
        └ by factor 2

8. Prove or disprove $f(x) = 2x^3 + 5$, $0 \le x \le 2$ is a one-to-one function, algebraically.

Suppose there are numbers X_1 and X_2 such that

$f(X_1) = f(X_2)$

$\Rightarrow 2X_1^3 + 5 = 2X_2^3 + 5$

$\Rightarrow 2X_1^3 = 2X_2^3$

$\Rightarrow X_1^3 = X_2^3$

$\Rightarrow X_1 = X_2$ ∴ f is one-to-one

9. Given the equation $x^2 + y^3 - x^2y^2 = 64$, find a) x-intercept(s), b) y-intercept(s), and c) test for symmetry about the x-axis.

· X-Intcp $(y=0)$: $x^2 + 0 - 0 = 64$

$$X = \pm 8$$

∴ $(8, 0), (-8, 0)$

· y-Intcp $(x=0)$: $0 + y^3 - 0 = 64$

$$\Rightarrow y^3 = 64$$

$$\therefore y = 4$$

∴ $(0, 4)$

· symmetry about the x-axis
  plug in $-y$ for y.

$$\Rightarrow x^2 + (-y)^3 - x^2(-y)^2 = 64$$

$$\Rightarrow x^2 - y^3 - x^2y^2 = 64$$

not equal to original
equation

∴ not symmetry

a) x-intercept(s): $(8, 0), (-8, 0)$
b) y-intercept(s): $(0, 4)$
c) symmetry about the x-axis: No

Question	Points Awarded	Question	Points Awarded
Multiple Choice 1-10			
Work out 1		6	
2		7	
3		8	
4		9	
5			
		Total	