MATH 150, FALL 2014
EXAM III WORK OUT - VERSION A

LAST NAME(print): ______________________ FIRST NAME(print): ______________________

UIN: ______________________ SECTION NUMBER: ____________

DIRECTIONS:

1. This is a 10-question work-out exam; Each problem is worth 5 points for a total of 50 points. Write all solutions in the space provided as full credit will not be given without complete, correct accompanying work, even if the final answer is correct.
2. Fully simplify all your answers, and give exact answers unless otherwise stated.
3. Circle your final answer.
4. The use of a calculator and computer is prohibited.
5. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
6. Your exam grade (sum of both exam parts) will be posted in WebAssign.
7. You may not discuss the contents of the exam with anyone until the exam is returned in class.

THE AGGIE CODE OF HONOR

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Signature: ______________________________

My signature in this blank allows my instructor to pass back my graded exam in class or allows me to pick up my graded exam in class on the day the exams are returned. If I do not sign the blank or if I am absent from class on the day the exams are returned, I know I must show my Texas A&M student ID during my instructor's office hours to pick up my exam.

Signature: ______________________________

1. Find the a) domain, b) range, and c) x-intercept(s) of $f(x) = 2e^{x+2} - 1$. If there is none, writh "None".

X-Intercept(s) $(y=0)$: $2e^{x+2} - 1 = 0$

$\Rightarrow 2e^{x+2} = 1$

$\Rightarrow e^{x+2} = \frac{1}{2}$

take ln

$\Rightarrow x+2 = \ln \frac{1}{2}$

$\Rightarrow x = \ln\frac{1}{2} - 2$

or $= -\ln 2 - 2$

a) Domain (interval notation): $(-\infty, \infty)$

b) Range (interval notation): $(-1, \infty)$

c) x-intercept(s): $\ln\frac{1}{2} - 2$ or $-\ln 2 - 2$

2. What are the a) vertical asymptote(s), b) horizontal asymptote and c) hole(s) as point(s), respectively, if any, of the function $f(x) = \dfrac{3x^2 - 12}{-2x^2 - 4x}$. If none, write "none".

$$f(x) = \frac{3\cancel{(x+2)}(x-2)}{-2x\cancel{(x+2)}}$$

restrictions : $x=0,\ x=-2$

$$= \frac{3(x-2)}{-2x}$$

$\Rightarrow$ hole at $x=-2$

$\Rightarrow$ hole is $(-2, f(-2))$

$= (-2, -3)$

$\Rightarrow$ V.A : $x = 0$

H.A : $y = -\frac{3}{2}$

a) Vertical asymptote(s): $x=0$

b) Horizontal asymptote: $y = -3/2$

c) Hole(s): $(-2, -3)$

3. Evaluate and fully simplify the following

(a) $\log_3 100 - \log_3 18 - \log_3 50$

$= \log_3 \frac{\cancel{100}\ 2}{\cancel{18}_9 \cdot \cancel{50}} = \log_3 \frac{1}{9} = \log_3 3^{-2} = -2$

(b) $\log_2 8^{22}$ $= \log_2 (2^3)^{22} = \log_2 2^{66} = 66$

(c) $\ln(\ln e^{e^{300}})$ $= \ln(e^{300}) = 300$

(d) $\log_9 \left(\frac{1}{27}\right)$ $= \log_{3^2} 3^{-3} = -\frac{3}{2}$

(e) $e^{2\ln 6}$ $= e^{\ln 6^2} = 6^2 = 36$

4. The fox population in a certain region has a relative growth rate of 8 % per year. It is estimated that the population in 2003 was 100. How many years after 2003 with the population reach 240?

$$P(t) = 100 \cdot e^{0.08t}$$

Find t which satisfies $P(t) = 240$

$$\Rightarrow 100 \cdot e^{0.08t} = 240$$

$$\Rightarrow e^{0.08t} = \frac{12}{5}$$

take ln.

$$\Rightarrow 0.08t = \ln\frac{12}{5}$$

$$\boxed{\therefore t = \frac{\ln\frac{12}{5}}{0.08} \text{ years.}} \quad \text{or} \quad t = \frac{\ln 2.4}{0.08}.$$

5. Exactly evaluate $\sin\left(\frac{7}{12}\pi\right)$. $\rightarrow$ $\frac{7\pi}{12} = \frac{7\pi}{\cancel{12}_{1}} \cdot \frac{\cancel{180°}^{15}}{\cancel{\pi}} = 105°$

$$= 60° + 45°$$

$$\Rightarrow \sin\left(\frac{7\pi}{12}\right) = \sin(105°) = \sin(60° + 45°)$$

$$= \sin 60° \cdot \cos 45° + \cos 60° \cdot \sin 45°$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}.$$

6. If $f(x) = \dfrac{2x^3 + 2x^2 - 4x}{x^4 - x}$, find a) the domain. b) x-intercept(s), c) y-intercept, d) horizontal asymptote, and e) vertical asymptote(s). If there is none, write "None".

$$f(x) = \frac{2x(x^2+x-2)}{x(x^3-1)} = \frac{\cancel{2x}(x+2)\cancel{(x-1)}}{\cancel{x}\cancel{(x-1)}(x^2+x+1)}$$: restrictions : $x=0, 1$

$$= \frac{2(x+2)}{x^2+x+1}$$

x-Intercept(s) ($y=0$) $\Rightarrow \dfrac{2(x+2)}{x^2+x+1} = 0 \Rightarrow x+2=0 \quad \therefore x = -2$

a) Domain (interval notation): $(-\infty, 0) \cup (0, 1) \cup (1, \infty)$

b) x-intercept(s): -2

c) y-intercept: none

d) Horizontal Asymptote: $y=0$

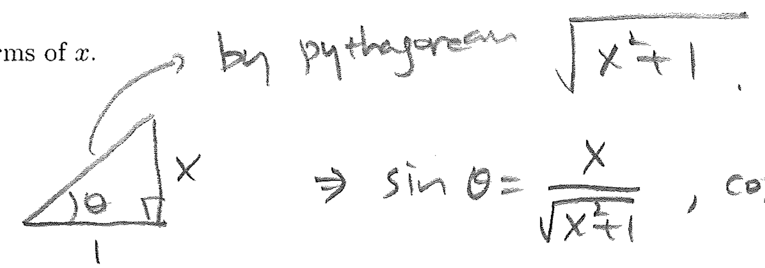
e) Vertical Asymptote(s): none

7. Use the generalized technique to calculate $\lim_{x\to\infty} \dfrac{10^9x^3 + 76x^2 - 2^5}{2x^4} \dfrac{/x^4}{/x^4} = 0$

$$= \lim_{x\to\infty} \frac{\cancel{\frac{10^9}{x}}^{\,0} + \cancel{\frac{76}{x^2}}^{\,0} - \cancel{\frac{2^5}{x^4}}^{\,0}}{2} = \frac{0}{2} = 0$$

8. If $\tan\theta = x$, express $\cos(2\theta)$ in terms of x.

$\tan\theta = x$ means



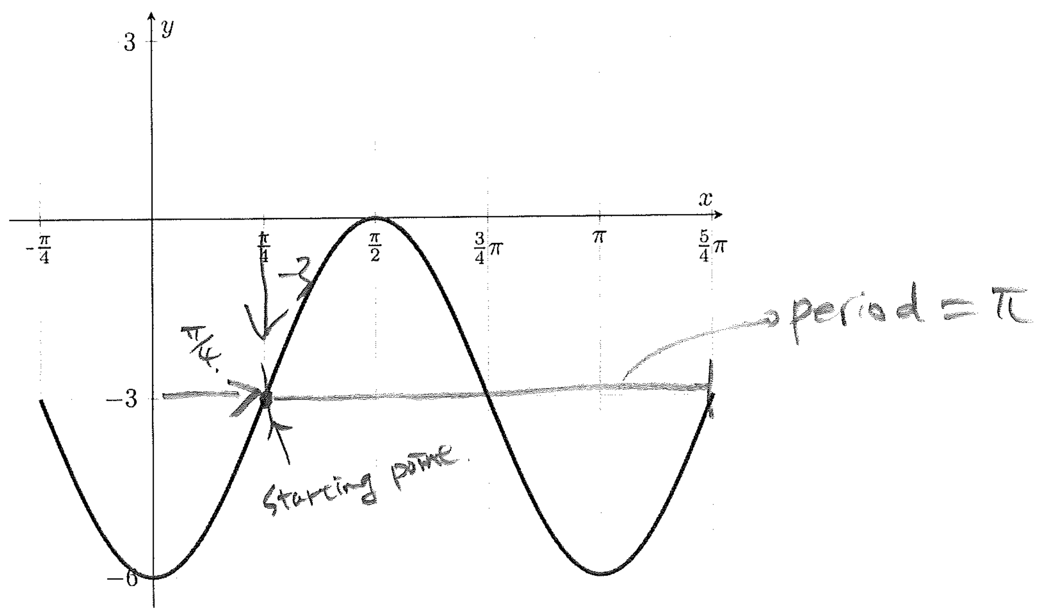
$\Rightarrow \sin\theta = \dfrac{x}{\sqrt{x^2+1}}$, $\cos\theta = \dfrac{1}{\sqrt{x^2+1}}$

since $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$$= \left(\frac{1}{\sqrt{x^2+1}}\right)^2 - \left(\frac{x}{\sqrt{x^2+1}}\right)^2$$

$$= \frac{1}{x^2+1} - \frac{x^2}{x^2+1} = \frac{1-x^2}{x^2+1}.$$

9. Write a function in the form of $f(x) = a\sin(k(x-b)) + c$, whose graph is shown below, where a, k and b are positive and as small as possible.



amplitude $= 3 \Rightarrow a = 3$

period $= \pi \Rightarrow k = 2$

phase shift $= \pi/4$

vertical shift $= -3$

$\Rightarrow f(x) = 3\cdot\sin\left(2\left(x - \frac{\pi}{4}\right)\right) - 3$

10. Prove the trigonometric identity: $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

Start from left-hand side,

$$\frac{2\tan x}{1+\tan^2 x} = \frac{2\,\frac{\sin x}{\cos x}}{1+\frac{\sin^2 x}{\cos^2 x}} = \frac{2\cdot\frac{\sin x}{\cos x}}{\frac{\cos^2 x+\sin^2 x}{\cos^2 x}} \quad =1$$

$$= \frac{2\cdot\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}} = \frac{\frac{2\sin x}{\cos x}}{\frac{1}{\cos^2 x}} = \frac{2\sin x\cdot\cos^{\cancel{2}} x}{\cancel{\cos x}}$$

$$= 2\sin x\cos x = \sin 2x.$$

Question	Points Awarded	Question	Points Awarded
Multiple Choice 1-10			
Work out 1		6	
2		7	
3		8	
4		9	
5		10	
		Total	