1. Prove that the triangle with vertices $A(-3,-2)$, $B(-2,2)$, and $C(6,0)$ is a right triangle.

pythagorean thm.

Note. $A(x_1,y_1)$, $B(x_2,y_2)$

$$d(A,B) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$d(A,B) = \sqrt{(-2-(-3))^2+(2-(-2))^2} = \sqrt{1^2+4^2} = \sqrt{1+16} = \sqrt{17}.$$

$$d(A,C) = \sqrt{(6-(-3))^2+(0-(-2))^2} = \sqrt{9^2+2^2} = \sqrt{81+4} = \sqrt{85} \text{ : hyp.}$$

$$d(B,C) = \sqrt{(6-(-2))^2+(0-2)^2} = \sqrt{8^2+2^2} = \sqrt{64+4} = \sqrt{68}$$

By Pythagorean.

$$(d(A,C))^2 = (d(A,B))^2 + (d(B,C))^2$$

$$(\sqrt{85})^2 = (\sqrt{17})^2 + (\sqrt{68})^2$$

$$85 = 17 + 68$$

$$= 85 \quad \checkmark$$

$\therefore \triangle ABC$ is right triangle.

Note. circle equation. $(x-x_1)^2+(y-y_1)^2=r^2$, center (x_1, y_1), radius $=r$

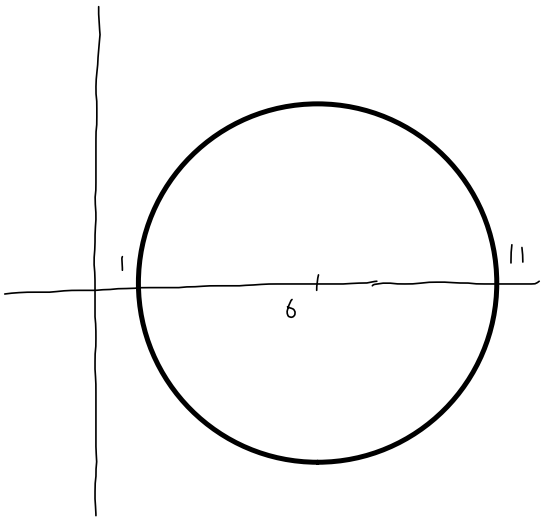
2. Write an equation that describes all the points (x, y) that are 5 units from point $B(6, 0)$.

Method 1) $(x-6)^2+(y-0)^2=5^2$

$\Rightarrow (x-6)^2+y^2=25$

Method 2) $\sqrt{(x-6)^2+(y-0)^2}=5$

$(x-6)^2+y^2=25$



domain : $[1, 11]$

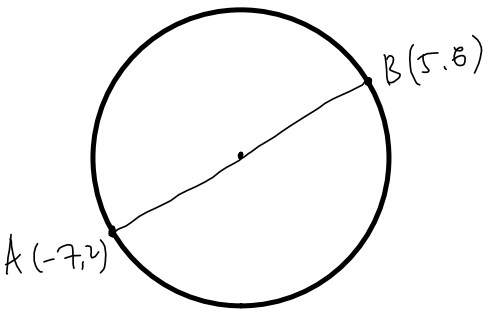
range : $[-5, 5]$

x-intercepts : $(1, 0), (11, 0)$

y-intercepts : none

Note midpoint $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

3. If $(-7, 2)$ and $(5, 8)$ are endpoints of a diameter of a circle, write the circle equation.



- center is midpoint of $\overline{AB}$.

$$M = \left(\frac{-7+5}{2}, \frac{2+8}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{10}{2}\right) = (-1, 5)$$

- radius $= \frac{d(A,B)}{2} = \frac{\sqrt{(5-(-7))^2+(8-2)^2}}{2}$

$$= \frac{\sqrt{144+36}}{2} = \frac{\sqrt{180}}{2}$$

180
2 90
2 45

$$= \frac{2\sqrt{45}}{2} = \sqrt{45}$$

$\therefore (x-(-1))^2 + (y-5)^2 = (\sqrt{45})^2$

$(x+1)^2 + (y-5)^2 = 45$

4. Find the center and radius for the following circles.

(a) $(x-3)^2+(y-7)^2=16$

center $(3,7)$, radius $=4$

(b) $(x+\frac{1}{2})^2+(y+5)^2=17$

center $\left(-\frac{1}{2},-5\right)$, radius $\sqrt{17}$

Note Completing the Square

$x^2+bx+\left(\frac{b}{2}\right)^2$ ✓

$= \left(x+\frac{b}{2}\right)^2$

$\therefore \left(x+\frac{b}{2}\right)^2=\left(x+\frac{b}{2}\right)\left(x+\frac{b}{2}\right)$

$=x^2+\frac{b}{2}x+\frac{b}{2}x+\frac{b^2}{2^2}$

$=x^2+bx+\frac{b^2}{2^2}$ ✓

(c) $x^2+y^2+8x+4=0$

$\Rightarrow x^2+8x+4^2+y^2=-4+4^2$

$\Rightarrow (x+4)^2+y^2=-4+16$

$=12$

$\therefore (x+4)^2+y^2=12$: center $(-4,0)$, radius $=\sqrt{12}=2\sqrt{3}$

(d) $2x^2+2y^2-8x+20y-2=0$

$\div 2$ $\div 2$

$\Rightarrow x^2+y^2-4x+10y-1=0$

$\Rightarrow x^2-4x+(-2)^2+y^2+10y+5^2=1+(-2)^2+5^2$

$(x-2)^2+(y+5)^2=1+4+25$

$=30$

$\therefore (x-2)^2+(y+5)^2=30$

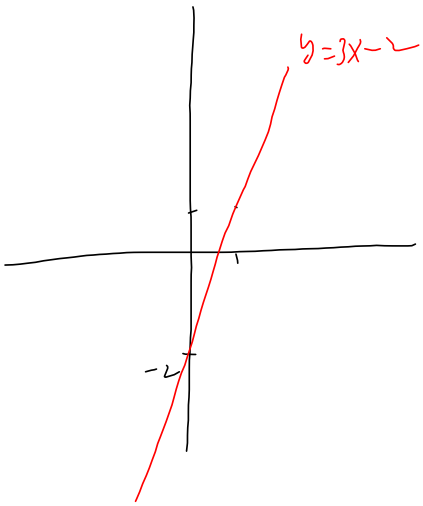
center : $(2,-5)$

radius $=\sqrt{30}$

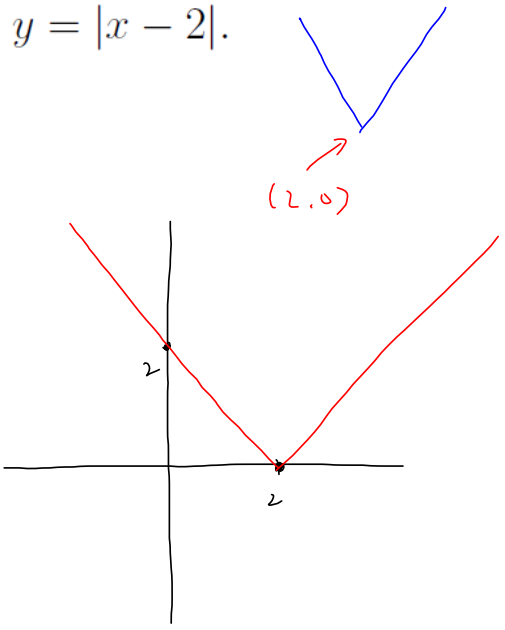
5. Graph $y = 3x - 2$.

y-int cp : $(0, -2)$

$(1, 1)$



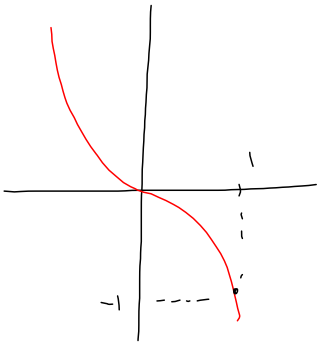
6. Graph $y = |x - 2|$.



Note $y = -|x|$

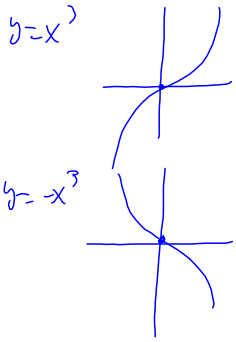
range : $y \geq 0$ or $[0, +\infty)$

7. Graph $y = -x^3$.



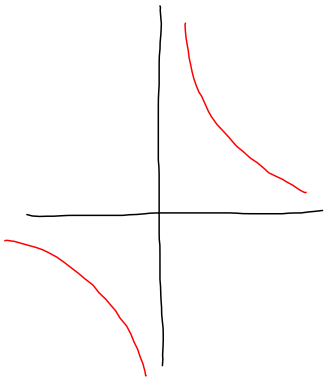
Note $y=x^3$

$y=-x^3$

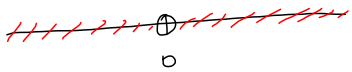


8. Graph $xy = 1$.

$y = \frac{1}{x}$



domain $\mathbb{R}$ except $x=0$



$\therefore (-\infty, 0) \cup (0, +\infty)$: domain, range

9. Find the intercepts for $|x| - y = 2$.

· X-intcps ($y=0$) : $|x| - 0 = 2$

$|x| = 2$

$\therefore x = 2$ or -2

$\therefore$ X-intcps are $(2,0)$, $(-2,0)$

· y-intcp ($x=0$) : $|0| - y = 2$

$-y = 2$

$\therefore y = -2$

$\therefore$ y-intcp is $(0,-2)$

10. Find the intercepts for $x^2 + 4x - 2xy = y + 5$.

· X-intcps ($y=0$) : $x^2 + 4x = 5$

$x^2 + 4x - 5 = 0$

$(x-1)(x+5) = 0$

$\therefore x = 1, -5$

$\therefore$ X-intcps : $(1,0)$, $(-5,0)$

· y-intcps ($x=0$) : $0 + 0 - 0 = y + 5$

$\therefore y = -5$

$\therefore$ y-intcp: $(0,-5)$

11. Test the equation for symmetry about the x-axis, y-axis, or origin.

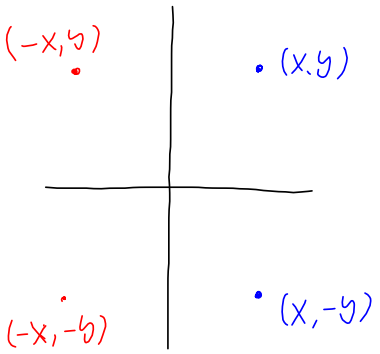
(a) $x^2 + y^2 - 6x = 2$

- X - axis (plug in $-y$ for y)

$x^2 + (-y)^2 - 6x = 2$

$x^2 + y^2 - 6x = 2$ ✓

$\therefore$ symmetry about x-axis



- y-axis (plug in $-x$ for x)

$(-x)^2 + y^2 - 6(-x) = 2$

$x^2 + y^2 + 6x = 2$ ✗

$\therefore$ not symmetry about y-axis

$\therefore$ not symmetry about the origin.

(b) $xy = x|x^2 - 1|$

- x-axis (plug in $-y$ for y)

$$x(-y) = x|x^2-1|$$

$$-xy = x|x^2-1|$$ ✗

∴ not symmetry about x-axis

- y-axis (plug in $-x$ for x)

$$(-x)\cdot y = (-x)|(-x)^2-1|$$

$$-xy = -x|x^2-1|$$

$$xy = x|x^2-1|$$ ✓

∴ symmetry about y-axis

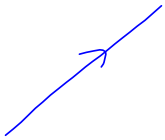
∴ not symmetry about the origin.

12. Find the slope of the line through each pair of points, and determine whether each line is increasing, decreasing, horizontal or vertical.

(a) $A(-2,1)$ and $B(3,5)$ $\quad x_1\ y_1 \quad x_2\ y_2$

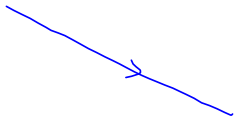
Note $\text{slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

$m = \frac{5-1}{3-(-2)} = \frac{4}{5}$: positive : increasing



(b) $C(0,-3)$ and $D(-4,1)$

$m = \frac{1-(-3)}{-4-0} = \frac{4}{-4} = -1$: negative : decreasing



(c) $E(3,5)$ and $F(3,-5)$

$m = \frac{-5-5}{3-3} = \frac{-10}{0}$: undefind : vereical line

(d) $G(0,-2)$ and $H(3,-2)$

$m = \frac{-2-(-2)}{3-0} = \frac{0}{3} = 0$: horizontal line

13. If the line through $(1, 5)$ and $(6, y_2)$ is parallel to the line $3x - 5y = 10$, find y_2.

$$m = \frac{y_2 - 5}{6 - 1} = \frac{y_2 - 5}{5}$$

$$-5y = -3x + 10$$

$$y = \frac{3}{5}x - 2$$

$$\therefore \text{slope} = \frac{3}{5}$$

$$\Downarrow$$

$$\frac{y_2 - 5}{5} = \frac{3}{5}$$

$$\Rightarrow y_2 - 5 = 3$$

$$\therefore y_2 = 8$$

Note. point-slope.
slope = m, point (x_1, y_1)
$y - y_1 = m(x - x_1)$

14. Write an equation for each of following lines.

(a) Line through $(0, -1)$ that is parallel to $3x + y = 2$.

$y = -3x + 2$
slope $= -3$

$y - (-1) = -3(x - 0)$

$y + 1 = -3x$

$y = -3x - 1$

(b) Line through $(0, 3)$ that is perpendicular to $y = -\frac{3}{4}x + 1$.

slope $= -\frac{3}{4}$
slope $= \frac{4}{3}$

$y - 3 = \frac{4}{3}(x - 0)$

$y - 3 = \frac{4}{3}x$

$\therefore y = \frac{4}{3}x + 3$

(c) Line through $(5, 2)$ that is perpendicular to $2x - y = 5$.

$-y = -2x + 5$

$y = 2x - 5$

slope $= 2$
slope $= -\frac{1}{2}$

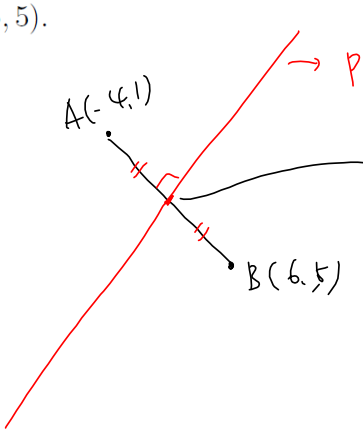
$\therefore y - 2 = -\frac{1}{2}(x - 5)$

$y - 2 = -\frac{1}{2}x + \frac{5}{2}$

$y = -\frac{1}{2}x + \frac{5}{2} + 2$

$= -\frac{1}{2}x + \frac{9}{2}$ $\quad \therefore y = -\frac{1}{2}x + \frac{9}{2}$

15. Write an equation for the perpendicular bisector of the line segment connecting $A(-4,1)$ and $B(6,5)$.



→ perpendicular bisector

midpoint of $\overline{AB} = \left(\frac{-4+6}{2}, \frac{1+5}{2}\right)$

$= \left(\frac{2}{2}, \frac{6}{2}\right) = (1, 3)$

slope of $\overline{AB} = \frac{5-1}{6-(-4)} = \frac{4}{10} = \frac{2}{5}$

$\Rightarrow$ slope $= -\frac{5}{2}$

$\therefore$ perpendicular bisector : $y - 3 = -\frac{5}{2}(x-1)$

$y - 3 = -\frac{5}{2}x + \frac{5}{2}$

$y = -\frac{5}{2}x + \frac{5}{2} + 3$

$= -\frac{5}{2}x + \frac{5}{2} + \frac{6}{2}$

$\therefore y = -\frac{5}{2}x + \frac{11}{2}$

16. For $f(x) = 4x - 3$, evaluate $\dfrac{f(a+h) - f(a)}{h}$.

$f(a+h) = 4(a+h) - 3$

$= 4a + 4h - 3$

$\Rightarrow \dfrac{f(a+h) - f(a)}{h} = \dfrac{4a+4h-3 - (4a-3)}{h}$

$= \dfrac{4a+4h-3-4a+3}{h}$

$= \dfrac{4h}{h} = 4$

17. For $f(x) = 3x^2 - 2x - 1$, evaluate $\dfrac{f(a+h) - f(a)}{h}$.

$f(a+h) = 3(a+h)^2 - 2(a+h) - 1$

$= 3(a^2 + 2ah + h^2) - 2(a+h) - 1$

$= 3a^2 + 6ah + 3h^2 - 2a - 2h - 1$

$\Rightarrow \dfrac{f(a+h) - f(a)}{h} = \dfrac{3a^2 + 6ah + 3h^2 - 2a - 2h - 1 - (3a^2 - 2a - 1)}{h}$

$= \dfrac{3a^2 + 6ah + 3h^2 - 2a - 2h - 1 - 3a^2 + 2a + 1}{h}$

$= \dfrac{6ah + 3h^2 - 2h}{h}$

$= \dfrac{h(6a + 3h - 2)}{h}$

$= 6a + 3h - 2$