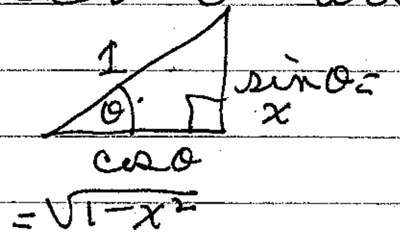
3.6 Examples (3.7 Ex. follow)

1) Find $\frac{d}{dx}\left(\cos(\arcsin x)\right)$

Simplify first:

Let $\theta = \arcsin x$ Then $\sin\theta = x$



From the triangle:

$$\cos(\arcsin x) = \cos\theta$$
$$= \sqrt{1-x^2}$$

$$\frac{d}{dx}\left(\sqrt{1-x^2}\right) = \frac{d}{dx}\left((1-x^2)^{1/2}\right)$$

$$= \frac{1}{2}(1-x^2)^{-1/2}(-2x) = \boxed{\frac{-x}{\sqrt{1-x^2}}}$$

2) Find $\frac{d}{dx}\left(\tan(\arcsin x)\right)$. Simplify first.

From the same triangle in 1) we see $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{\sqrt{1-x^2}}$

Using the quotient rule:

$$\frac{d}{dx}\left(\frac{x}{\sqrt{1-x^2}}\right) = \frac{\sqrt{1-x^2} - x\left(\frac{-x}{\sqrt{1-x^2}}\right)}{(1-x^2)}$$

$$= \frac{\sqrt{1-x^2} + x^2/\sqrt{1-x^2}}{(1-x^2)} \cdot \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}$$

$$= \frac{1}{(1-x^2)^{3/2}}$$

3) Find $\frac{d}{dx}\left(\arcsin(x^2)\right)$

Using the chain rule:

$$\frac{d}{dx}\left(\arcsin(x^2)\right) = \frac{1}{\sqrt{1-(x^2)^2}}(2x)$$

$$= \frac{2x}{\sqrt{1-x^4}}$$

4) Find $\frac{d}{dx}\left(x \arctan x\right)$.

Using the product rule:

$$\arctan x + \frac{x}{1+x^2}$$

5) Find $\frac{d}{dx}\left(\arccos e^{x}\right)$

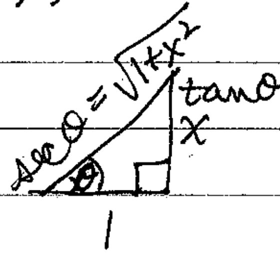
Using the chain rule:

$$\frac{-1}{\sqrt{1-(e^{x})^{2}}} \cdot e^{x} = \frac{-e^{x}}{\sqrt{1-e^{2x}}}$$

6) Find $\frac{d}{d\theta}\left(\arctan(\sin\theta)\right)$

$$= \frac{1}{1+\sin^{2}\theta}(\cos\theta) = \frac{\cos\theta}{1+\sin^{2}\theta}$$

7) Find $\frac{d}{dx}\left(\sec(\arctan x)\right)$ Simplify first.



Let $\theta = \arctan x$

$\tan\theta = x$

$\sec\theta = \sqrt{1+\tan^{2}\theta} = \sqrt{1+x^{2}}$

$$\frac{d}{dx}\left(\sqrt{1+x^{2}}\right) = \frac{x}{\sqrt{1+x^{2}}}$$

Examples 3.7

1) $\frac{d}{dx}\left(\log_2 x\right) = \frac{1}{x \ln 2}$

2) $\frac{d}{dx}\left(\log_3(1+\cos x)\right) = \frac{-\sin x}{1+\cos x}$

3) $\frac{d}{dx}\log_2\left(\frac{x+1}{x+2}\right) = \frac{d}{dx}\left(\log_2(x+1) - \log_2(x+2)\right)$

$= \frac{1}{(x+1)\ln 2} - \frac{1}{(x+2)\ln 2}$

Given $\ln\left[\frac{(f(x))^r (g(x))^s}{(h(x))^t}\right]$ for example, use log rules before taking the derivative.

Ex. $\frac{d}{dx}\left[\ln\left(\frac{x^2(x-2)^3}{(x^2+3)^4}\right)\right]$

$= \frac{d}{dx}\left[2\ln x + 3\ln(x-2) - 4\ln(x^2+3)\right]$

$= \frac{2}{x} + \frac{3}{x-2} - 4\,\frac{2x}{x^2+3}$

$= \frac{2}{x} + \frac{3}{x-2} - \frac{8x}{x^2+3}$

4. $\frac{d}{dx}\ln(\sec^2 x) = \frac{d}{dx}\left[2\ln|\sec x|\right]$

$= \frac{1}{\sec^2 x}\left[2\sec x \sec x \tan x\right]$

$= 2\tan x$

So $\frac{d}{dx}\left(\ln|\sec x|\right) = \tan x$

5. $\frac{d}{dx}\left(\ln|x|\right)$

$$\ln|x| = \begin{cases} \ln(-x) & x<0 \\ \ln x & x>0 \end{cases} \quad \text{undefined if } x=0$$

For $x<0$

$\frac{d}{dx}\left(\ln(-x)\right) = \frac{1}{-x}(-1) = \frac{1}{x}$.

$x>0$

$\frac{d}{dx}\left(\ln x\right) = \frac{1}{x}$ So $\frac{d}{dx}\left[\ln|x|\right] = \frac{1}{x}$

6. $\frac{d}{dx}\left[\ln(x^2+1)\right] = \frac{2x}{x^2+1}$

7. $\frac{d}{dx}\left[\ln(e^x+1)\right] = \frac{e^x}{e^x+1}$