Math 131 Final Exam Review Key

Additional Problems covering regression are on my website below the Final Exam Rev.

1. $A = -2 \quad B = 6 \quad C = \ln\frac{1}{2} = -\ln 2$

2. $A = 2 \quad B = 2 \quad C = 2$

3. $A = 4 \quad B = -2 \quad C = 3$

4. $x = 0$ or $x = 2$

5. $x = \ln 5$ Note: $\ln(-1)$ is undefined.

6. $x = \frac{22}{15}$

7. $\frac{1}{5}\ln\left(\frac{1897}{1500}\right) \approx .047$

8. $\boxed{r = -.0135}$ (The decay rate is .0135)

9. a) The limit does not exist at $x = -1$ since $x = -1$ is a vertical asymptote.

b) Not continuous at $x = -1$ since the limit DNE.

Since $\lim\limits_{x \to 1^-} f(x) = \lim\limits_{x \to 1^-} \frac{2}{1+x} = 1$ and

$\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1} x^2 = 1$ and $f(1) = 1^2 = 1$

f is continuous at $x = 1$.

9c) $\frac{d}{dx}\left[f(x)\right]_{x<1} = \frac{d}{dx}\left[\frac{2}{x+1}\right]_{\substack{x \neq -1 \\ x<1}} = \frac{-2}{(x+1)^2}$

The slope of the left side is $\frac{-2}{(1+1)^2} = -\frac{1}{2}$ at $x=1$.

$\frac{d}{dx}\left[f(x)\right]_{x>1} = \frac{d}{dx}\left[x^2\right] = 2x$

The slope of the right side is $2(1)=2$ at $x=1$.

The two sides meet at different slopes making a corner at $x=1$.

$\boxed{\text{Not differentiable at } x=1.}$

10. $f'(x) = \frac{1}{3}(x^2-25)^{-2/3}(2x) = \frac{2x}{3(x^2-25)^{2/3}}$

which does not exist at $x=-5$ or at $x=5$.

11. a) $\left.\frac{d}{dx}\left[\ln x\right]\right|_{x=2} = \frac{1}{2}$ b) $\left.\frac{d}{dx}\left[e^x\right]\right|_{x=0} = e^0 = 1$

c) $\frac{d}{dx}\left[5x^3 - 8x^2 + 3\right] = 15x^2 - 16x$

12. $\frac{2}{x} + \frac{8x}{x^2+4} - \frac{12x^2+28}{x^3+7x}$

13 a) $y = 54(x-1) + 27$

b) $y = 1$

c) $y = \frac{1}{4}x + 1/4$

14. $y = -20x + 52$

15. a) local max at $x = -4/5$
local min at $x = 0$
inflection points at $x = -2$,
$x = \frac{-4+\sqrt{6}}{5}$ and $x = \frac{-4-\sqrt{6}}{5}$

b) local min at $x = 1/2$ no local max
inflection pts. at $x = 1$ and $x = 2$.

c) local min's at $x = 0$ and $x = 2$
local max at $x = 1$
inflection pts. at $x = 1 - \sqrt{1/3}$ and
$x = 1 + \sqrt{1/3}$

d) local min at $x = 0$. no local max.
inflection pts. at $x = -\frac{2}{\sqrt{3}}$ and
$x = \frac{2}{\sqrt{3}}$

16. $df = \frac{1}{3}x^{-2/3}\,dx$

$\sqrt[3]{8.5} - \sqrt[3]{8} \approx df$ when $x = 8$ $dx = 8.5 - 8 = .5$

$= \frac{1}{3}(8)^{-2/3}(.5) = \frac{1}{24}$

so $\sqrt[3]{8.5} \approx \sqrt[3]{8} + \frac{1}{24} = 2 + 1/24$

The tangent line is $y = \frac{1}{12}(x-8) + 2$

The tangent line approximation to $f(8.5)$ is

$y(8.5) = \frac{1}{12}(8.5-8) + 2 = 2 + \frac{1}{24}$ (same as above)

17. \$1.375 or \$1.38

18. The partions $\perp$ to the wall are $5\sqrt{10}$ ft.

The side // to the wall is $\frac{200}{\sqrt{10}}$ ft.

19. The sides $\perp$ to the wall are $10\sqrt{15}$ ft.

The partitions // to the wall are $\frac{100}{\sqrt{15}}$ ft.

20.

20. $L_4 = 3.75 \quad R_4 = 7.5$

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n = \int_0^1 16^x\,dx = \frac{16^x}{\ln 16}\Big|_0^1$$

$$= \frac{16-1}{\ln 16} \approx 5.41$$

21. a) $a(t) = v'(t) = \dfrac{1-\ln(1+t)}{(1+t)^2}$

b) $\displaystyle\int_0^4 v(t)\,dt = \frac{1}{2}\left[\ln(1+t)\right]^2\Big|_0^4 \approx 1.295$

22. a) $2e-2$ b) 587.5 c) $\frac{1}{2}\ln 2$

23. a) 10 b) $\frac{34}{3}$

24. a) $v(r) = 16400(10^{-4} - r^2)$

b) Flux $= 8.2\pi \times 10^{-5}$