Math 131 In-Class Exam I Review
Solutions

1a) The domain of $\frac{x+1}{x-2}$ is $(-\infty, 2) \cup (2, \infty)$ and e^u is defined for all u so the domain is $\boxed{(-\infty, 2) \cup (2, \infty)}$

b) $x^2+4 \geq 4 > 0$ for all x so the domain is all real numbers, $\boxed{(-\infty, \infty)}$

c) x^2-4 must be positive. Looking at the graph, this is true on $\boxed{(-\infty, -2) \cup (2, \infty).}$

d) $\ln(x-2)$ is defined for $x>2$, $\ln(x+2)$ is defined for $x>-2$, So both are defined for $x>2$ or on $\boxed{(2, \infty)}$

Note: $\ln(x^2-4) = \ln(x-2) + \ln(x+2)$ wherever both are defined, that is on $(2, \infty)$.

e) $\sqrt[3]{u}$ is defined for all u and x^2-25 is defined for all x the domain is $\boxed{(-\infty, \infty)}$.

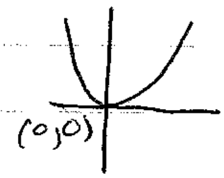
2. x = # of pages copied. For $0 \le x \le 100$,
$f(x) = 0.25x$ dollars
For $x > 100$, $f(x) = 0.25(100) + 0.2(x-100)$
$= 25 + 0.2x - 20$
$= 5 + 0.2x$ dollars

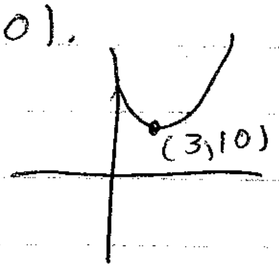
$$f(x) = \begin{cases} 0.25x & 0 \le x \le 100 \\ 5 + 0.2x & 100 < x \end{cases}$$

3. Since $f(x)$ is a transformation of x^2 without an x-distortion, we know

$f(x) = a(x-h)^2 + k$ a y-distortion, possibly a reflection, a horiz. shift and a vertical shift.

The vertex of the parabola is $(3,10)$ which is the transf. of $(0,0)$.
$h = 3$ and $k = 10$





Find $\underline{a}$: Plug in $(2, 15)$
$f(2) = a(2-3)^2 + 10 = a + 10 = 15$
so $a = 5$

$$\boxed{f(x) = 5(x-3)^2 + 10}$$

4. a) The vertical asymptote of $y = \ln x$ is $x = 0$ which has been shifted to $x = -4$ so $C = 4$.

$f(x) = A + B\ln(x+4)$

Plug in the given pts. to solve for A and B:

$e - 4 + 4 = e$ and $\ln e = \log_e e = 1$ so

$f(e-4) = A + B = 3$

$e^2 - 4 + 4 = e^2$ and $\ln e^2 = 2$ so

$f(e^2-4) = A + 2B = 0$

$$\begin{array}{r} A + 2B = 0 \\ A + B = 3 \\ \hline B = -3 \end{array}$$

subtract

Substitute $B = -3$ into

$A + B = 3$

$A + (-3) = 3 \quad A = 6$

$f(x) = 6 - 3\ln(x+4)$

4 b) $A = -2$ $\quad A + Be^{c \cdot 0} = -2 + B = 0$

$B = 2$

$f(x) = -2 + 2e^{cx}$

$f(1) = -2 + 2e^{c} = -1$

$2e^{c} = 1 \quad e^{c} = \frac{1}{2}$

$c = \ln(\frac{1}{2})$

$f(x) = -2 + 2e^{x\ln(\frac{1}{2})} = -2 + 2e^{\ln(\frac{1}{2})^x}$

$= \boxed{-2 + 2\left(\frac{1}{2}\right)^x}$

5. a) $e^{-2x} = (e^{-x})^2$

so $f(g(x)) = (e^{-x})^2 + 5e^{-x} + 1$

$g(x) = e^{-x}$ and $f(x) = x^2 + 5x + 1$

b) $g(x) = x^2 + 3$ and $f(x) = \ln x$

6. a) $N(t) = 200(1.5)^{t/3}$

b) $(1.5)^{t/3} = e^{\ln(1.5)^{t/3}} = e^{t\frac{\ln 1.5}{3}}$

$N(t) = 200e^{t\left(\frac{\ln 1.5}{3}\right)}$

7. The quartic fits the given data a little better but turns downward past $x=90$. Since stamp prices will probably continue to rise, the exponential would be better for extrapolating.

8a) Swithing x and y and solving for y:

$x = 5e^{3y}$ $\quad \frac{x}{5} = e^{3y}$ $\quad \ln\frac{x}{5} = 3y$

$y = \frac{1}{3}\ln\left(\frac{x}{5}\right) = \frac{1}{3}\ln x - \frac{1}{3}\ln 5 = f^{-1}(x)$

b) $x = \ln(2y-5)$ $\quad e^x = 2y-5$

$e^x + 5 = 2y$

$f^{-1}(x) = \frac{1}{2}e^x + \frac{5}{2}$

9. a) $x = e^{\ln(2^4)} = 2^4 = 16$

b) $\sqrt{4} = 2$

$4^{-\frac{1}{2}} = \frac{1}{2}$ $\quad \boxed{x = -\frac{1}{2}}$

c) $\log_2(x+3) - \log_2(x+4) = 3$; $\log_2\left(\frac{x+3}{x+4}\right) = 3$

$\frac{x+3}{x+4} = 2^3 = 8$

$x+3 = 8x+32$ $\quad -29 = 7x$

$\frac{-29}{7} = x$ so $\boxed{\text{no solution}}$

We cannot plug $-\frac{29}{7}+3$ or $\frac{-29}{7}+4$ into $\log_2 u$.

10. Find $d(5)$ and $d(2)$:

$$d(5) = 162 - 2(25) = 112$$
$$d(2) = 162 - 2(4) = 15$$
$$\frac{d(5)-d(2)}{5-2} = \frac{112-154}{3} = -\frac{42}{3} = -14\ \frac{ft}{sec}$$

is v_{avg}.

Find v_{inst} at 2 sec:

$$\frac{d(t)-d(2)}{t-2} = \frac{162-2t^2-154}{t-2} = \frac{8-2t^2}{t-2}$$

$$= \frac{2(4-t^2)}{t-2} = \frac{2(2-t)(2+t)}{t-2} \qquad \frac{2-t}{t-2} = -1$$

$$= -2(t+2) \xrightarrow[t \to 2]{} -2(2+2)$$
$$= -8\ ft/sec$$

The negative means it is going left.

[illegible]
the start.

11. a) Substitution of $x=1$ gives "$\frac{0}{0}$".
Factoring we have:

$$\frac{2(x^2+2x-3)}{(x-1)(x+1)} = \frac{2(x+3)(x-1)}{(x-1)(x+1)} = \frac{2(x+3)}{x+1} \text{ if } x \neq 1$$

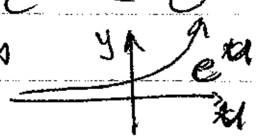
$$\lim_{x\to 1} f(x) = \frac{2(1+3)}{1+1} = \frac{8}{2} = \boxed{4}.$$

b) Since substituting -1 gives "$\frac{\text{nonzero}}{0}$", the limit $\boxed{\text{DNE}}$

c) Substituting $x=-3$ gives $\boxed{0}$.

12. a) First find $\lim_{x\to 0^-} \frac{1}{x} = -\infty$

$\lim_{u\to -\infty} e^u = 0$ so $\boxed{\lim_{x\to 0^-} e^{1/x} = 0}$



b) $\lim_{x\to 0^+} \frac{1}{x} = +\infty$ so $\lim_{x\to 0^+} e^{1/x} = +\infty$

c) $$\frac{4x^5-15x^2}{2x^2-5} \sim \frac{4x^5}{2x^2} = 2x^3 \xrightarrow[x\to\infty]{} \boxed{\infty}$$

$\sim$ means "has the same limit as"

d) Same as c except $x \to -\infty$ and $2x^3$ is negative for $x<0$ so the limit is $\boxed{-\infty}$

12e) Both go to ∞ and "$\infty - \infty$" is indeterminate.

So conjugate:

$$\sqrt{x^2-15x} - \sqrt{x^2+2x} = \frac{(x^2-15x)-(x^2+2x)}{\sqrt{x^2-15x}+\sqrt{x^2+2x}}$$

$$= \frac{-17x}{\sqrt{x^2-15x}+\sqrt{x^2+2x}} \sim \frac{-17x}{2x} \xrightarrow[x\to\infty]{} \boxed{\frac{-17}{2}}$$

12 f) $\dfrac{\sqrt{x}+1}{-3\sqrt{x}+2} \sim \dfrac{\sqrt{x}}{-3\sqrt{x}} \xrightarrow[x\to\infty]{} \boxed{-1/3}$

13. a) f is continuous for all x. No discont.

since $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} e^{1/x} = 0$ and $\sin 0 = 0 = f(0)$

and $\lim\limits_{x\to 0^+} \sin x = 0$.

b) $x^2-1=(x-1)(x+1)$ We need to check $x=-1$ and $x=1$.

Factor numerator: $2(x^2+2x-3) = 2(x+3)(x-1)$

$g(x) = \dfrac{2(x+3)(x-1)}{(x+1)(x-1)} = \dfrac{2(x+3)}{x+1}$ if $x<1$

$\lim\limits_{x\to -1} g(x)$ DNE and g has a V.A. $x=-1$

$$\lim_{x \to 1^-} g(x) = \frac{2(1+3)}{1+1} = 4$$

$$\lim_{x \to 1^+} g(x) = 4(1) = 4 = g(1)$$

So the only discontinuity is at -1.