

## Math 131 In Class Exam 2 Review Solutions

1. The limit definition of the derivative function is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

For  $f(x) = e^x + x^2$ , this is

$$f'(x) = \lim_{h \rightarrow 0} \frac{e^{(x+h)} + (x+h)^2 - (e^x + x^2)}{h}$$

2. Tangent line:  $y = f'(a)(x-a) + f(a)$

$$a) f'(x) = 3x^{-1/4} - 2$$

$$f'(16) = 3(16^{-1/4}) - 2 = \frac{3}{2} - 2 = -\frac{1}{2}$$

$$f(16) = 4(16^{3/4}) - 2(16) + 9 \\ = 4(8) - 32 + 9 = 9$$

$y = -\frac{1}{2}(x-16) + 9$  is the tangent line.

$$b) f'(x) = e^x + \frac{1}{x} \quad f'(1) = e + 1$$

$$f(1) = e + \ln 1 = e$$

$y = (e+1)(x-1) + e$  is the tangent line

3. Look for V.A.'s of  $f'$  which make V.T.'s in  $f$ , corners, and any discontinuities.

$$a) f(x) = x^{2/5} \quad f'(x) = \frac{2}{5} x^{-3/5} = \frac{2}{5} \cdot \frac{1}{x^{3/5}}$$


$f'$  has a V.A. @  $x=0$  so  $f$  has a V.T. that is  $f'$  DNE at  $x=0$ .

Note:  $f$  is continuous for all  $x$ .

$$b) f(x) = (x^2 - 9)^{2/5} \quad f \text{ is continuous for all } x$$

$$f'(x) = \frac{2}{5} (x^2 - 9)^{-3/5} (2x) = \frac{4x}{5(x^2 - 9)^{3/5}}$$

At  $x = -3$  and  $x = 3$ ,  $f'$  DNE (V.A.)  
 $f$  has V.T.

c)  $|x|$  has a corner at  $x=0$    
so  $f$  is not differentiable at  $x=0$ .

Check continuity at 1.  $\checkmark \lim_{x \rightarrow 1} |x| = 1 = \lim_{x \rightarrow 1} x^2 = f(1)$

$x^2$  is differentiable everywhere and  $6(x-3)+12$  is also.

Is  $f$  continuous at  $x=3$ ?  $9 = 3^2 = \lim_{x \rightarrow 3^-} f(x)$

but  $\lim_{x \rightarrow 3^+} f(x) = 6(3-3)+12 = 12$  Not Cont.

So  $f$  is not differentiable at  $x=3$

Question:

If we make continuous by changing the right piece to  $6(x-3)+9$ , then is  $f$  differentiable? slope of  $x^2$  at 3 is  $2x|_{x=3} = 6$  and the slope of  $6(x-3)+9$  is 6 so yes.  $x=3$

4.  $h(1) = f(g(1))$  since the tangent to  $g$  at 1 is  $y = -x + 4$ ,  $g(1) = -1 + 4$  and  $g'(1) = -1$ .

$$g(1) = y(1) = -1 + 4 = 3 \quad \boxed{h(1) = f(3) = 3^2 - 2 = 7}$$

$$g'(1) = -1 \quad h'(1) = f'(g(1))g'(1) \text{ Chain Rule}$$

$$= f'(3)g'(1).$$

$$f'(u) = 2u \quad \text{so } f'(3) = 6$$

$$\boxed{h'(1) = 6(-1) = -6}$$

$$y = -6(x-1) + 7 \text{ is the tangent to } h \text{ at } x=1.$$

5. Product Rule  $(f \cdot g)' = f'g + fg'$

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$$

$$a) \frac{d}{dx} \left[ \frac{x}{e^{2x} + 1} \right] = \frac{e^{2x} + 1 - x(e^{2x} \cdot 2)}{(e^{2x} + 1)^2}$$

$$= \frac{e^{2x} + 1 - 2xe^{2x}}{(e^{2x} + 1)^2}$$

$$b) f(x) = \frac{x^2}{x} + \frac{4x}{x} - \frac{7}{x} = x + 4 - 7x^{-1}$$

$$f'(x) = 1 + 7x^{-2} = 1 + \frac{7}{x^2}$$

5c) Rewrite  $f(x)$  as

$$3 \ln(x^2+4) + \frac{1}{4} \ln x - \ln(e^x+1)$$

$$f'(x) = 3 \cdot \frac{2x}{x^2+4} + \frac{1}{4} \cdot \frac{1}{x} - \frac{e^x}{e^x+1}$$

$$= \frac{6x}{x^2+4} + \frac{1}{4x} - \frac{e^x}{e^x+1}$$

$$d) \frac{d}{dx} [\cos(\pi x)] = -\pi \sin(\pi x)$$

$$e) \frac{d}{dx} [\sin^2 x] = \frac{d}{dx} [(\sin x)^2]$$

$$= 2 \sin x \cos x$$

$$f) \frac{d}{dx} [\tan(x^2)] = 2x \sec^2(x^2)$$

$$g) \frac{d}{dx} [\sec(3x)] = 3 \sec(3x) \tan(3x)$$

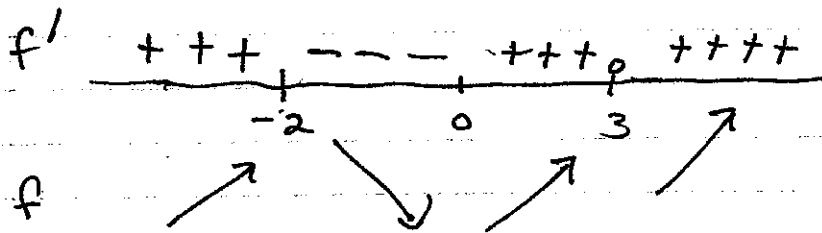
$$6. a) f(x) = (x+2)^2 (x-3)^3$$

$$f'(x) = 2(x+2)(x-3)^3 + 3(x+2)^2(x-3)^2$$

$$= (x+2)(x-3)^2 [2(x-3) + 3(x+2)]$$

$$= (x+2)(x-3)^2 [2x-6 + 3x+6]$$

$$= (x+2)(x-3)^2 5x$$



local max  
at  $x = -2$

local min  
at  $x = 0$

$$f'(x) = (x+2)(x-3)^2 5x = (5x^2 + 10x)(x-3)^2$$

$$f''(x) = (10x+10)(x-3)^2 + (5x^2+10x) \cdot 2(x-3)$$

$$= (x-3) \left( (10x+10)(x-3) + (5x^2+10x) \cdot 2 \right)$$

$$= (x-3) (10x^2 - 20x - 30 + 10x^2 + 20x)$$

$$= (x-3) (20x^2 - 30)$$

$$= (x-3) \cdot 10(2x^2 - 3)$$

$$= 10(x-3)(\sqrt{2}x - \sqrt{3})(\sqrt{2}x + \sqrt{3})$$

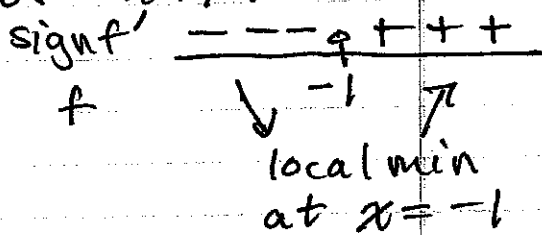
inflection pts. at  $x = 3$ ,  $x = \frac{\sqrt{3}}{\sqrt{2}}$  and  $x = -\frac{\sqrt{3}}{\sqrt{2}}$

$f''$  - , + , - , +

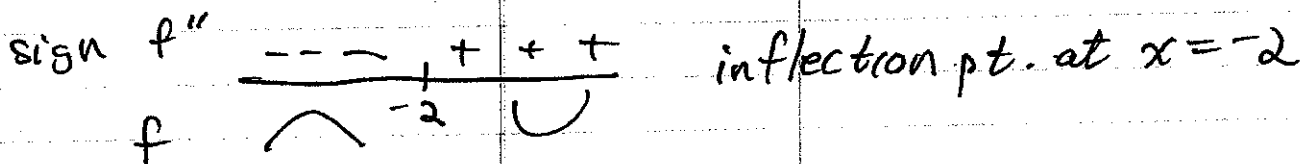
6b)  $f(x) = xe^x$

$f'(x) = e^x + xe^x = (1+x)e^x$

$e^x > 0$  for all  $x$  so the sign chart for  $f'$  is the same as the sign chart for  $x+1$ .



$f''(x) = e^x + (1+x)e^x = (2+x)e^x$



7.  $v(t)$  is positive and  $a(t)$  is positive.

...  $v(t)$  has a <sup>local</sup>  $v_{max}$ ,  $a(t)$  is 0 and  $f$  has

an inflection pt.. Note:  $a(t)$  must change sign for  $v$  to have a local max.

If  $g' < 0$  and decreasing then  $g$  is decreasing and concave down.

$$8. a) f'(t) = \frac{1}{(\arcsin t)} \cdot \frac{-1}{\sqrt{1-t^2}}$$

$$b) f'(t) = \frac{1}{1+(e^t)^2} \cdot e^t = \frac{e^t}{1+e^{2t}}$$