Math 131 InClass Exam 3 Review
Solutions

1. a) We know $\sqrt[4]{16} = 2$. Let $f(x) = \sqrt[4]{x}$.

Then $f(17) - f(16) = \sqrt[4]{17} - 2 \approx df$

$df = f'(x)\,dx = \frac{1}{4}x^{-3/4}\,dx$

Plug in $x = 16$ and $dx = 17 - 16 = 1$

$$df = \frac{1}{4}(16^{-3/4}) \cdot 1 = \boxed{\frac{1}{32}} \approx f(17) - f(16)$$
$$= \sqrt[4]{17} - 2$$

b) $\sqrt[4]{15} - 2 \approx df$ for $x = 16$ and $dx = 15 - 16 = -1$

$$df = \frac{1}{4}(16^{-3/4})(-1) = \frac{-1}{32} \approx \sqrt[4]{15} - 2$$

so $\sqrt[4]{15} \approx \boxed{2 - 1/32}$

c) $\sqrt[4]{16.5}$ $\quad df = \frac{1}{32}(.5) = \frac{1}{64}$

$$\sqrt[4]{16.5} \approx \boxed{2 + \frac{1}{64}}$$

2. $f(x) = x^3 - 3x^2$ is continuous and $[-2, 3]$ is a closed bounded interval.
Find the critical values in $[-2, 3]$:

$f'(x) = 3x^2 - 6x$

$= 3x(x-2)$ c.v.'s $x = 0$ $x = 2$

x	-2	0	2	3
$f(x)$	-20	0	-4	0

Absolute min is -20 at $x = -2$.
Absolute max is 0 at $x = 0$ and $x = 3$.

3. $\dfrac{e^x}{x^2}$ has a discontinuity at $x = 0$.

$\lim\limits_{x \to 0^-} \dfrac{e^x}{x^2} = +\infty = \lim\limits_{x \to 0^+} \dfrac{e^x}{x^2}$ so there is no absolute max on $[-1, 1]$.

There is an absolute min on $[-1, 1]$:

$$\frac{d}{dx}\left(\frac{e^x}{x^2}\right) = \frac{x^2 e^x - 2x e^x}{x^4} = \frac{x e^x (x-2)}{x^4}$$

$$= \frac{e^x (x-2)}{x^3}$$

$f'(x)$ + | − | +

$\nearrow$ 0 $\searrow$ 2 $\nearrow$

Comparing $f(-1) = e^{-1}$ and $f(1) = e$

we see the absolute min on $[-1, 1]$ is e^{-1}

3 b) $\frac{e^x}{x^2}$ is continuous on $[1, 3]$

so it will have an absolute max and absolute min on $[1, 3]$.

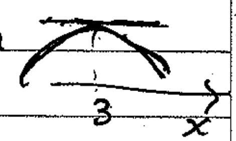
x	1	2	3
$f(x)$	e	$\frac{e^2}{4}$	$\frac{e^3}{9}$

Absolute min is $\frac{e^2}{4}$ at $x=2$

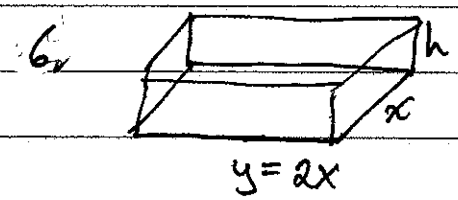
Absolute max is e at $x=1$
(Remember $e \approx 2.718\ldots < 3$)

4. $x=-1$ f is increasing.
$x=0$ f is increasing and concave down.
$x=1$ horizontal tangent, concave up so
f has a <u>local min</u> at $x=1$.

$x=2$ No conclusion
$x=3$ horizontal tangent, concave down
f has a local <u>max</u> at $x=3$



5 a) $v(a)=0$ and $a(a)>0$ <u>local min</u>
b) $v(a)=0$ and $a(a)<0$ <u>local max</u>



$$V = 2x^2 h = 10 \qquad h = \frac{10}{2x^2} = \frac{5}{x^2}$$

base cost $= 10 \cdot 2x^2$ (\$10/sqm $\cdot$ $2x^2$ sqm)

2 sides cost $= 2 \cdot 6 \cdot xh$

front and back cost $= 2 \cdot 6 \cdot (2xh)$

$$\text{Total cost} = 20x^2 + 12xh + 24xh$$
$$= 20x^2 + 36xh$$
$$= 20x^2 + 36x \cdot \frac{5}{x^2} \qquad (h = 5/x^2)$$

$$C(x) = 20x^2 + \frac{180}{x}$$

$$C'(x) = 40x - \frac{180}{x^2} \qquad \text{Set } C'(x) = 0,$$

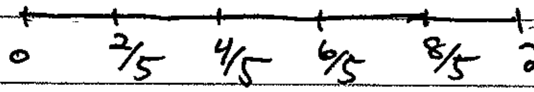
$$40x = \frac{180}{x^2} \qquad 40x^3 = 180$$
$$x^3 = \frac{18}{4} = 4.5$$

$$x = \sqrt[3]{4.5} \qquad y = 2\sqrt[3]{4.5}$$

$$h = \frac{5}{(4.5)^{2/3}}$$

7. $\frac{b-a}{n} = \frac{2}{5}$ = base of each rectangle.

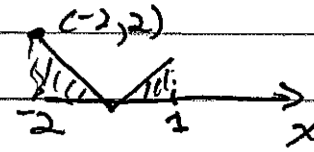
Partition pts:



$$L_5 = \frac{2}{5}\left[\underset{1}{e^0} + e^{2/5} + e^{4/5} + e^{6/5} + e^{8/5}\right]$$

$$R_5 = \frac{2}{5}\left[\ e^{2/5} + e^{4/5} + e^{6/5} + e^{8/5} + e^2\right]$$

8. a) $\int_{-2}^{1} |x|\,dx$

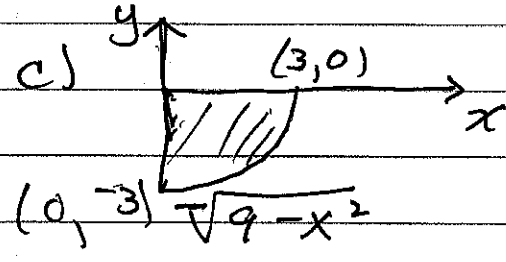


The Δ on the left has area $\frac{1}{2}(2\times 2) = 2$
The Δ on the right has area $\frac{1}{2}(1\times 1) = \frac{1}{2}$
so

$$\int_{-2}^{1} |x|\,dx = 2 + \frac{1}{2} = 2.5$$

b) Only odd powers of x appear in $f(x)$ so f is odd. The interval is symmetric about $x=0$ so $\int_{-30}^{30} f(x)\,dx = 0$.

c)



$-\frac{1}{4}$ area of circle of radius 3

$= -\frac{1}{4}\cdot 9\pi$

$= -\frac{9\pi}{4}$

9. $f''(t) = 8e^{2t}$

$f'(t) = 8\left(\frac{1}{2}e^{2t}\right) + c = 4e^{2t} + c$

Since $f'(0) = 4e^{0} + c = -6$

$4 + c = -6$

$c = -10$

$f(t) = 2e^{2t} - 10t + d$

Since $f(0) = 3$

$2e^{0} - 0 + d = 3$

$2 + d = 3 \quad d = 1$

$\boxed{f(t) = 2e^{2t} - 10t + 1}$

10. a) $\int x^{-2/3}\,dx = 3x^{1/3} + C$

b) $6e^{t} - 4\cos t + C$

c) $\dfrac{x^3 - 2x}{\sqrt{x}} = x^{3-1/2} - 2x^{1-1/2}$

$= x^{5/2} - 2x^{1/2}$

$\int x^{5/2} - 2x^{1/2}\,dx = \frac{2}{7}x^{7/2} - \frac{4}{3}x^{3/2} + C$

d) $\int \sec^2 t\,dt = \tan t + C$

11.

a) Let $u = x^2 - 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int x(x^2-1)^8\,dx = \int \frac{1}{2}u^8\,du = \frac{1}{18}u^9 + C$$

$$= \frac{1}{18}(x^2-1)^9 + C$$

b) $x^3 = x^2 \cdot x$ Use one x to be part of du.
Solve for $x^2 = u+1$ for the x^2 part.

$u = x^2 - 1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$$\int \frac{1}{2}(u+1)u^8\,du = \int \frac{1}{2}u^9 + \frac{1}{2}u^8\,du$$

$$= \frac{1}{20}u^{10} + \frac{1}{18}u^9 + C$$

$$= \frac{1}{20}(x^2-1)^{10} + \frac{1}{18}(x^2-1)^9 + C$$

c) Let $u = 3e^x + 1$

$du = 3e^x\,dx$

$\frac{1}{3}du = e^x\,dx$

$$\int \frac{1}{3}u^{1/2}\,du$$

$$= \frac{1}{3}\cdot\frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{9}(3e^x+1)^{3/2} + C$$

d) Let $u = \ln x$

$du = \frac{1}{x}dx$

$$\int u\,du = \frac{1}{2}u^2 + C$$

$$= \frac{1}{2}(\ln x)^2 + C$$

12. a) Let $u = x+5 \longrightarrow u-5 = x \to u-3 = x+2$

$du = dx$

$$\int \frac{x+2}{\sqrt{x+5}}\,dx = \int \frac{u-3}{\sqrt{u}}\,du = \int \frac{u}{\sqrt{u}} - \frac{3}{\sqrt{u}}\,du$$

$$= \int u^{1/2} - 3u^{-1/2}\,du = \frac{2}{3}u^{3/2} - 6u^{1/2} + C$$

$$= \frac{2}{3}(x+5)^{3/2} - 6(x+5)^{1/2} + C$$

$$\int_{-1}^{4} \frac{x+2}{\sqrt{x+5}}\,dx = \frac{2}{3}(x+5)^{3/2} - 6(x+5)^{1/2}\Big|_{-1}^{4}$$

$$= \frac{2}{3}(9^{3/2}) - 6(9^{1/2}) - \left[\frac{2}{3}(4^{3/2}) - 6(4^{1/2})\right]$$

$$= 18 - 18 - \left(\frac{16}{3} - 12\right) = \frac{20}{3}$$

Alternatively:

$$\int_{-1}^{4} \frac{x+2}{\sqrt{x+5}}\,dx \underset{u=x+5}{=} \int_{4}^{9} \frac{u-3}{\sqrt{u}}\,du$$ (Doing the subst. work)

$$= \frac{2}{3}u^{3/2} - 6u^{1/2}\Big|_{4}^{9} = \frac{20}{3}$$

12 b) $\int_0^{\ln 6} e^x(e^x+3)^{1/2}\,dx$

$u = e^x + 3 \qquad u(0) = e^0 + 3 = 4$

$du = e^x\,dx \qquad u(\ln 6) = e^{\ln 6} + 3 = 6+3 = 9$

$$\int_4^9 u^{1/2}\,du = \frac{2}{3}u^{3/2}\Big|_4^9 = \frac{2}{3}(27) - \frac{2}{3}(8)$$

$$= 18 - \frac{16}{3} = \frac{38}{3}$$