Math 131 Exam 1 Review

1. Find the domain of each function, \( f(x) = \)
   \( a) \sqrt{100 - x^2} \quad b) \frac{1}{\sqrt{100 - x^2}} \quad c) \sqrt{x^2 - 9} \quad d) \ln(x + 2) \quad e) \ln x^2 \quad f) \ e^{x^2 - 3x} \quad g) \sqrt{x^2 - 3x + 2} \)

2. A tax schedule requires you to pay 15\% of the first $20000 of income plus 20\% of the amount above $20000. For example a person earning $50000 pays 15\% of $20000 plus 20\% of $30000. Write the tax as a piecewise function of income, \( x \).

3. \( f(x) \) is a transformation of \( y = x^2 \). The range of \( f \) is \([ -\infty, 12] \), \( f(-3)=12 \) and \( f(0)=-6 \). You do not need a horizontal distortion. Write the formula for \( f(x) \).

4. \( f(x) \) is a transformation of \( y=\ln(x) \). \( f(x) \) approaches minus infinity as \( x \) approaches 2, \( f(3)=7 \) and \( f(e+2)=10 \). Write a formula for \( f(x) \) assuming no \( x \)-distortion was done.

5. Solve for \( A, B \) and \( C \) if \( f(x) = A + Be^{Cx} \)
   \( f(x) \rightarrow 3 \ as \ x \rightarrow \infty \quad f(0) = 8 \quad f(1) = 7 \)

   List the transformations of \( y = e^x \), in a proper order, that result in \( f(x) \).

6. A culture begins with 100 cells and grows exponentially so that after 2 hours, there are 250 cells. Find \( N(t) \), the number of cells after \( t \) hours.

7. Give two possibilities for \( f \) and \( g \) if \( f(g(x)) = (x^2 - 2x + 1)^{1/3} \).

8. An account earning continuous compound interest had the values shown at ends of the first through the fifth years.
   \[
   \begin{array}{cccccc}
   \text{year} & 1 & 2 & 3 & 4 & 5 \\
   \text{account value} & 1056.54 & 1116.28 & 1179.40 & 1246.08 & 1316.58 \\
   \end{array}
   \]

   a) Do an exponential regression and write the exponential model.
   Write the model in base \( e \) using \( b = e^{\ln b} \).
   Estimate the annual interest rate and the initial principal.

   b) According to the model, what is the approximate doubling time? tripling time?

   c) If the \( y \) values in the table were replaced by \( \ln y \), which model would best fit?

   d) If the \( x \) and \( y \) values in the table were switched, which model would best fit?
9. Solve for x if:  
   a) \( e^{2x} + 2e^x + 1 = 16 \)  
   b) \( e^{2x} - 3e^x + 2 = 0 \)

10. Solve for \( x \) exactly.

   a) \( e^{-3\ln 2} = x \)
   b) \( \log_4(x+12) - 5\log_4\sqrt{2} = 3 \)
   c) \( \log_4(x+3) + \log_4(x-3) = 2 \)

   d) \( \log_4(x^2 - 9) = 2 \)
   e) \( \log_3(3x+2) - \log_3(x-2) = 3 \)

Section 2.1 problem

11. A particle travels in a straight line. The directed distance from a reference point is given by \( d(t) \) ft. where \( t \) is in seconds. Find the average velocity for \( t \) between 1 and 3 secs. Find the instantaneous velocity at \( t=1 \) sec.

   a) \( d(t) = 8t + 12 \)
   b) \( d(t) = 60t - 16t^2 \)
   c) \( d(t) = 15 - \sqrt{t} \)

12. Evaluate each limit or state DNE if it does not exist.

   a) \( \lim_{x \to 4} \frac{x^2 + x - 20}{x^2 - 16} \)
   b) \( \lim_{x \to -4} \frac{x^2 + x - 20}{x^2 - 16} \)
   c) \( \lim_{x \to -5} \frac{x^2 + x - 20}{x^2 - 16} \)

   d) \( \lim_{x \to 0} x \ln(x^2) \)
   e) \( \lim_{x \to 0^+} e^{1/x} \)
   f) \( \lim_{x \to 0^+} e^{1/x} \)
   g) \( \lim_{x \to \pi/2} \frac{\sin^2 x - 1}{\sin x - 1} \)

13. Find the value of \( c \) which makes \( f(x) \) continuous for all \( x \).

   \[ f(x) = \begin{cases} 
   x^2 + cx - 1 & \text{if } x < 2 \\
   7x & \text{if } 2 \leq x 
   \end{cases} \]

14.

   \[ f(x) = \begin{cases} 
   \sin \frac{x}{x} & \text{if } x < 0 \\
   1 & \text{if } x = 0 \\
   x \ln x & \text{if } 0 < x 
   \end{cases} \]

   Find a) \( \lim_{x \to 0^+} f(x) \) and b) \( \lim_{x \to 0^-} f(x) \).

   Is \( f \) left continuous at 0, right continuous at 0, both or neither?
15. Find all discontinuities of \( f(x) \) and give either a graphical or a definition reason for each discontinuity.

\[
f(x) = \begin{cases} 
\frac{x^2 + 5x + 6}{x^2 + 2x} & x < 1 \\
x^2 + 3 & 1 \leq x \leq 5 \\
\frac{5x + 11}{x + 1} & 5 < x 
\end{cases}
\]

Section 2.5 problem
16. Find each limit as a number, infinity or minus infinity, or state DNE.

\[
a) \lim_{{x \to \infty}} e^{\frac{1}{x}} \quad b) \lim_{{x \to \infty}} e^{\frac{x+1}{x-2}} \quad c) \lim_{{x \to \infty}} \frac{x^5 - 4x^4 + 27}{x^2 + 2x} \\
d) \lim_{{x \to \infty}} \frac{x^4 + 2x^3}{x^2 + 10} \quad e) \lim_{{x \to \infty}} \frac{5x^3 - 15x^2 + 3}{7x^3 + 25} \\
f) \lim_{{x \to \infty}} \left(\sqrt{x^2 + 7x} - \sqrt{x^2 - 3x}\right)
\]

17. Find the inverse function for each.

\[
a) \quad f(x) = 4e^{x+2} \quad b) \quad f(x) = \ln(7x + 3)
\]