1. Use sign charts to find the local max, local min and inflection point(s) and the interval where the function is concave up for each function.

a) \( f \) is continuous except at \( x = -2 \) and \( f'(x) = \frac{(x - 1)^3}{(x + 2)^2} \).

b) \( f \) is continuous and defined for \( x > 0 \) and \( f'(x) = \frac{\ln x}{x^2} \).

c) \( f(x) = xe^{-2x} \)

d) \( f(x) = \sin(2x) \) on \([0, \pi]\)

2. Assume \( f'' \) is continuous. The table shows values of \( f' \) and \( f'' \) at 4 different values of \( x \). Give the conclusion of the 2nd derivative test at each \( x \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( x_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f'(x) )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( f''(x) )</td>
<td>0</td>
<td>-2</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

3. \( f(x) = (x^2 + 1)g(x) \)  \( g(0) = 0 \)  \( g'(0) = 0 \)  \( g''(0) = -1 \)

and \( g'' \) is continuous. According to the 2nd derivative test, \( g(x) \) has a local ______ at \( x=0 \) and \( f(x) \) has a local ______ at \( x=0 \).

4. Given that \( f(x) \) is continuous on \([a, b]\), which statement(s) is/are true?

i) \( f(x) \) must have a local max and a local min on \([a, b]\).

ii) \( f(x) \) must have an absolute max and an absolute min on \([a, b]\).

iii) If \( c \) is the only critical value in \((a, b)\) and \( f \) has a local min at \( c \) then it has an absolute min at \( c \).
5. a) A cherry grower has an orchard in which the trees currently yield 3 lbs of cherries each. The yield per tree is increasing by 0.4 lbs per day. The current price he can get for the cherries is $4/lb and this is decreasing by $0.20 /day. How many days should he wait to pick his cherries to maximize his revenue per tree? Hint: revenue per tree is yield per tree times price per lb. Let \( d \) = the number of days he waits and find yield per tree as a function of \( d \) and find price/lb as a function of \( d \).

b) When should he pick them if his yield/tree is only increasing by 0.1lb/day but all else is the same as in a?

6. A fence must enclose a 1200 sq ft rectangular area. The area will be divided by a partition parallel to a side. Find the dimensions of the rectangle that will minimize the amount of fencing required. Verify that you found the dimensions for the minimum.

7. A rectangular area must be enclosed by 600 ft of fencing. If a building will form one side of the fence, what dimensions will maximize the area? Verify you found the max.

8. A vehicle uses 20 gal of gas for a certain 600 mile trip if the average speed is 80 mph. The vehicle uses 15 gal of gas for the same trip if the average speed is 55 mph. Assume a linear relation between the gallons of gas and average speed. The cost of gas is $3.50/gallon. Other costs of the trip total $7/hour. Find the average speed that minimizes the total cost of the trip. Show you found the minimum.

9. Find the most general antiderivative.

\[ a) \quad f(x) = \frac{(x + 2)^2}{x} \quad b) \quad f(x) = \frac{x^3 + 1}{\sqrt{x}} \quad c) \quad f(x) = \sin x \cos x \]

For c you can substitute \( u = \sin x \) or \( u = \cos x \). How do the answers differ?

\[ d) \quad f(x) = \frac{(\ln x)^4}{x} \quad e) \quad f(x) = \frac{e^{\sqrt{x}}}{\sqrt{x}} \quad f) \quad f(x) = (x^2 + 5)e^{x^3 + 15x + 4} \]

\[ g) \quad f(x) = (x^2 + 2x)\sqrt{x^3 + 3x^2 + 15} \quad h) \quad f(x) = x\sqrt{x - 2} \]
10. An object travels forward and backward along a straight path. The velocity at time \( t \) is

\[
v(t) = \begin{cases} 
\sqrt{4 - t^2} & 0 \leq t \leq 2 \\
2 - t & 2 < t
\end{cases} \text{ m/s}
\]

If \( s(0) = 0 \), find \( s(5) \), the position at 5 seconds.

Find the total distance traveled in the first 5 seconds.

\[
v(t) = \begin{cases} 
4 & 0 \leq t \leq 4 \\
12 - 2t & 4 < t \leq 8
\end{cases} \text{ m/s}
\]

Find \( s(8) \), the position at 8 seconds, assuming \( s(0) = 0 \).

Find the total distance traveled in the first 8 seconds.

11. Find \( F(x) \).

\[
a) \quad F'(x) = 6x^2 + \frac{3}{x} \quad F(1) = -1
\]

\[
b) \quad F''(x) = e^x \quad F(0) = 5 \quad F'(0) = 6
\]

\[
c) \quad F''(x) = \frac{10}{\sqrt[3]{x}} \quad F(1) = 6 \quad F(8) = 304
\]

12. Find the left hand and right hand Riemann sums and their average. Sketch the rectangles for each. Compare

\[
\frac{L_n + R_n}{2} \quad \text{and} \quad \lim_{n \to \infty} (L_n) = \lim_{n \to \infty} (R_n) = \int_a^b f(x) \, dx.
\]

\[
a) \quad f(x) = x^3 - 4x \quad \text{on} \quad [0,3] \quad n = 6
\]

\[
b) \quad f(x) = 9xe^{-x^2} \quad \text{on} \quad [0,1] \quad n = 3
\]

\[
c) \quad f(x) = 0.25(16^x) \quad \text{on} \quad [0,1] \quad n = 4
\]