When you are allowed to open the test, print your name on the inside page also.

There are 10 multiple choice and 4 work-out questions.

Each multiple choice question is 4 points. Each work-out question is 15 points.

Circle your multiple choice answers and fill in your scantron. Show all work in the work-out section.

"On my honor, I have neither given nor received unauthorized help on this exam."

Sign __________________________
1. A line contains the point $(5, 3)$. On this line, each increase of 2 in $x$ produces a decrease of 12 in $y$. Find the $y$-intercept of this line.

\[ m = \frac{\Delta y}{\Delta x} = \frac{-12}{2} = -6 \quad y - 3 = -6(x - 5) \]

\[ y = -6x + 33 \]

2. Choose the function that is the result of $y = e^x$ after applying the following transformations in the given order: reflection across the $y$-axis, shift left 3 units, stretch vertically by a factor of 2.

\[ 3) \quad y = 2e^{-2(x+3)} \]

\[ a) \quad y = -e^{2(x-3)} \quad b) \quad y = -e^{2x+3} \]

\[ c) \quad y = 2e^{-x+3} \quad d) \quad y = \frac{1}{2}e^{-x-3} \quad e) \quad y = 2e^{-(x+3)} \]

3. A substance weighed 20g at time $t=0$ days. It grows exponentially so that at $t=6$ it weighs 30g. At how many days will it weigh 50g? Round the number of days to 2 decimal places if needed.

\[ a) \quad 18 \quad b) \quad 5.61 \quad c) \quad 12.24 \quad d) \quad 13.56 \quad e) \quad 20 \]

\[ \text{Solve} \quad 20 \left( \frac{30}{20} \right)^{\frac{t}{6}} = 50 \quad \text{for} \quad t. \quad \Rightarrow \quad \frac{t}{6} \ln \left( \frac{3}{2} \right) = \ln \left( \frac{5}{2} \right) \quad \Rightarrow \quad t = \frac{6 \ln \left( \frac{5}{2} \right)}{\ln \left( \frac{3}{2} \right)} \approx 13.56 \]

4. The half-life of a drug in the body is 3 days. What portion of the original amount taken is present after 12 days?

\[ a) \quad \frac{1}{16} \quad b) \quad \frac{1}{8} \quad c) \quad \frac{1}{4} \quad d) \quad \frac{2}{3} \quad e) \quad \frac{1}{12} \]

\[ 12 = 4 \times 3 \quad \text{so it halves 4 times to get} \quad \frac{1}{16} \quad \text{of the original amount.} \]
5. Use your calculator to evaluate \( \lim_{x \to 0} \frac{\sin 7x}{2x} \). The limit is

\[ a) \ 1 \quad b) \ 0 \quad c) \ \frac{5}{3} \quad d) \ \frac{7}{2} \quad e) \ DNE \]

6. \( \lim_{x \to -3^+} -2\ln(x+3) \) is

\[ a) \ \infty \quad b) \ -\infty \quad c) \ 1 \quad d) \ 0 \quad e) \ -2 \]

7. \( f(x) = \begin{cases} 5x^2 + Ax & x < 2 \\ 3x + A & 2 \leq x \end{cases} \)

Find the value of \( A \) that makes \( f(x) \) continuous for all \( x \).

\[ a) \ 2 \quad b) \ -17 \quad c) \ 5 \quad d) \ 12 \quad e) \ -14 \]

So we have

\[ 5(2^2) + A(2) = 3(2) + A \]
\[ 20 + 2A = 6 + A \]
\[ A = -14 \]
8. \( f(x) = 5(2^x) - 6 \) Which is equal to \( f^{-1}(74) \)?

\begin{align*}
(a) \quad 4 & \quad b) \quad 0 & \quad c) \quad 16 & \quad d) \quad -\infty \\
\end{align*}

**Solve:**

\[ 5(2^x) - 6 = 74 \quad \text{for } x. \]

\[ 5(2^x) = 80 \]

\[ 2^x = 16 \quad x = 4 \]

9. \( f(x) = \ln(a \cdot b^x) \) can be simplified to

\begin{itemize}
  \item a) a line with slope \( \ln(a) \).
  \item b) a line with slope \( \ln(b) \).
  \item c) an exponential function.
  \item d) a power function.
\end{itemize}

\[ \frac{1}{a \cdot b^x} \]

By logarithm rules,

\[ \ln(a \cdot b^x) = \ln(a) + \ln(b^x) = \ln(a) + x \ln(b) \]

10. For \( f(x) = \ln(x^2 + 1) \) and \( g(x) = e^x \), \( h(x) = g(f(x)) \) is equal to

\begin{itemize}
  \item a) \( \ln(e^{2x} + 1) \)
  \item b) \( 2x + 1 \)
  \item c) \( 2x \)
  \item d) \( x^2 + 1 \)
  \item e) \( \ln(e^{x^2} + 1) \)
\end{itemize}

\[ g(f(x)) = e^{\ln(x^2 + 1)} = x^2 + 1 \]
Work Out Section:

I] a) Find the equation of the line through the points \((1, 2)\) and \((5, 7)\).

\[
m = \frac{7-2}{5-1} = \frac{5}{4}
\]

\[
y - 2 = \frac{5}{4}(x - 1)
\]

\[
y = \frac{5}{4}x - \frac{5}{4} + 2
\]

\[
y = \frac{5}{4}x + \frac{3}{4}
\]

b) For the data points

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

write the linear regression model.

\[
y = x + 0.8
\]

c) Find the cubic regression model. Write the leading coefficient rounded to 3 decimal places.

The leading coefficient is the coefficient of \(x^3\) which is about \(0.417\)

d) What does the cubic model predict for the \(y\)-value when \(x=6\)?

\[16.8\]

e) Viewing the graph, which two data points are closest to the cubic regression model?

\((1, 2)\) and \((5, 7)\)
II] A substance initially weighed 30 g but decays exponentially. After six years it weighs 20 g.

a) What will it weigh after 12 years?

$$30 \left( \frac{20}{30} \right)^{\frac{t}{6}} = 30 \left( \frac{20}{30} \right)^2 = \frac{40}{3} \text{ g}$$

$$t = 12 \quad 30 \left( \frac{4}{9} \right) =$$

b) Write the equation for the weight in grams after $t$ years.

$$A(t) = 30 \left( \frac{2}{3} \right)^{\frac{t}{6}} \quad \text{or} \quad 30 e^{\frac{t}{6} \ln \left( \frac{2}{3} \right)}$$

(either one)

c) Find the half-life of the substance, rounded to 3 decimal places if necessary.

Solve $\left( \frac{2}{3} \right)^{\frac{t}{6}} = \frac{1}{2}$

$$\ln \left( \frac{2}{3} \right)^{\frac{t}{6}} = \ln \frac{1}{2}$$

$$\frac{t}{6} \ln \left( \frac{2}{3} \right) = \ln \frac{1}{2}$$

$$t = \frac{6 \ln \frac{1}{2}}{\ln \left( \frac{2}{3} \right)} \approx 10.257 \text{ years}$$
III] Evaluate each limit as a number, as infinity or minus infinity, or state DNE.

For a and b, \( f(x) = \frac{x^2 - x - 6}{x^2 - 5x + 6} = \frac{(x - 3)(x + 2)}{(x - 2)(x - 3)} \)

a) \( \lim_{x \to 3^+} f(x) = \frac{3+2}{3-2} = 5 \)

b) \( \lim_{x \to 2^-} f(x) = -\infty \)

c) \( \lim_{x \to \infty} \frac{\sqrt{4x^3 + 5x}}{\sqrt{9x^3 + 25}} = \lim_{x \to \infty} \sqrt{\frac{4x^3 + 5x}{9x^3 + 25}} = \sqrt{\frac{4}{9}} = \frac{2}{3} \)

d) \( \lim_{x \to -\infty} \frac{7 + e^x}{5e^x - 10} \rightarrow \lim_{x \to -\infty} e^{-x} = 0 \)

So \( \frac{7 + e^x}{5e^x - 10} \rightarrow \frac{7 + 0}{0 - 10} = -\frac{7}{10} \)
IV] 
\[ f(x) = \begin{cases} 
\ln(2-x) & x < 2 \\
\frac{x^2 - 7x + 10}{x^2 - 4x - 5} & 2 \leq x < 7, \ x \neq 5 \\
3x & 7 \leq x \text{ or } x = 5 \\
\frac{(x-2)(x-5)}{(x+1)(x-5)} & x = 5
\end{cases} \]

Evaluate each of the following:

\[ \lim_{x \to 2^-} f(x) = -\infty \quad \lim_{x \to 2^+} f(x) = 0 \quad f(2) = 0 \]

V. A. \( x = 2 \)

Is \( f \) continuous at \( x = 2 \)? If not, why not? No. V. A. \( x = -2 \)

\[ \lim_{x \to 5^-} f(x) = \frac{5-2}{5+1} = \frac{3}{6} = \frac{1}{2} \quad \lim_{x \to 5^+} f(x) = \frac{1}{2} = \frac{1}{5} \]

Is \( f \) continuous at \( x = 5 \)? If not, why not? No. Hole at \( x = 5 \).

\[ \lim_{x \to 7^-} f(x) = \frac{7-2}{7+1} = \frac{5}{8} \]

Is \( f \) continuous at \( x = 7 \)? If not, why not? No. \( \lim f(x) \) DNE at \( x = 7 \). There is a gap/jump since the left and right hand limits are not equal.