Additional Topic from 4.3

The 2nd Derivative Test:

If f'' is continuous on (a, b) and $a < c < b$ and

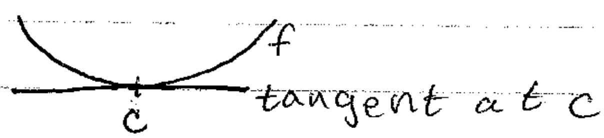
1) $f'(c) = 0$ $f''(c) > 0$ then f has a local min at c.

2) $f'(c) = 0$ $f''(c) < 0$ then f has a local max at c.

Explanation and Memory aid:

1) $f'(c) = 0$ says the tangent line is horizontal.

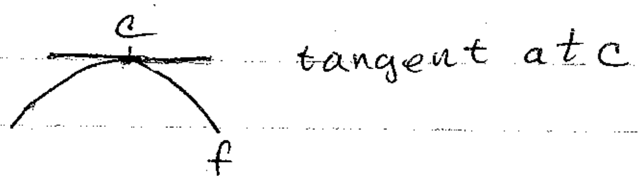
$f''(c) > 0$ says f is concave up.



f must have a local min @ c.

2) $f'(c) = 0$ so the tangent is horizontal.

$f''(c) < 0$ so f is concave down.



f must have a local max @ c.

3) If $f'(c) = 0$ and $f''(c) = 0$, no conclusion.

Example: Find local extrema for

$f(x) = x(\ln x)^2$ on $(0, \infty)$

$f'(x) = (\ln x)^2 + 2x(\ln x) \cdot \frac{1}{x}$

$= (\ln x)^2 + 2\ln x$

Find the critical pt(s).:

Solve $(\ln x)^2 + 2\ln x = 0$

$= (\ln x)[\ln x + 2]$

$\ln x = 0$ if $x = 1$ $\quad \ln x + 2 = 0$

if $\ln x = -2$

$x = e^{-2}$

Test each critical value using the 2nd derivative test:

$f''(x) = \frac{d}{dx}\left[(\ln x)^2 + 2\ln x\right]$

$= \frac{2\ln x}{x} + \frac{2}{x} = \frac{2}{x}(\ln x + 1)$

$f''(1) = 2 > 0$

f has a local min at $x = 1$.

$f''(e^{-2}) = 2e^2(-2+1) = -2e^2 < 0$

f has a local max at e^{-2}.