141 Final Exam Review w Key Solutions

1. t = age in years y = car value
$(0, 24{,}000)$ $(3, 16{,}500)$ are two points on the line.

$$\text{Slope} = \frac{16{,}500 - 24000}{3 - 0} = \frac{-7{,}500}{3} = -2{,}500$$

$$y - 24{,}000 = -2{,}500(t - 0) = -2{,}500t$$

$$y = -2500t + 24000$$

or observe $b = 2400$ since it is paired with $t = 0$.

2. $C(x) = \text{add'l cost}/\text{unit} \cdot x + \text{fixed cost}$
$R(x) = \text{price}/\text{unit} \cdot x$ $P(x) = R(x) - C(x)$

a) $C(x) = 5x + 30{,}000$
$R(x) = 8x$
$P(x) = 8x - 5x - 30000 = 3x - 30{,}000$

Break even when $P(x) = 0$.

b) $3x - 30{,}000 = 0$ if $x = 10{,}000$

3. a) $(0, 500)$ $(300, 710)$ $m = \frac{210}{300} = .7$

$\boxed{p = .7x + 500}$

b) $(200, 600)$ $\frac{\Delta p}{\Delta x} = \frac{25}{-50} = -.5$

Pt. Slope $y - y_1 = m(x - x_1)$

$y - 600 = -.5(x - 200) = -.5x + 100$

$\boxed{p = y = -.5x + 700}$

c) Set demand price equal to supply price.

$.7x + 500 = -.5x + 700$

$1.2x = 200$ $x = \frac{200}{1.2} = \frac{1000}{6} \cong 167$

4. STAT
1 EDIT

L1	L2
0	57
2	65.8
4	74.2
6	83.3

STAT ▶ CALC 4
$y = 4.365x + 56.98$

In 1991, $x = 1991 - 1983 = 8$

$y = 4.365(8) + 56.98 = 91.9$ billion

5. see key

6. $.8[A - B]$ or $A - B - .2(A - B)$

7. $\longrightarrow \downarrow$ $AB = \begin{bmatrix} b+2a & 1-a \\ 3b & 3 \\ 2b+b & 1 \end{bmatrix}$

8. $$\begin{array}{c|cc} & \text{city} & \text{country} \\ \hline \text{gas} & .04 & .03 \\ \text{hybrid} & .02 & .015 \end{array} \quad \begin{array}{c|c} \text{city} & 150 \\ \text{country} & 50 \end{array}$$

$$\text{or} \quad \begin{bmatrix} 150 & 50 \end{bmatrix} \begin{bmatrix} .04 & .02 \\ .03 & .015 \end{bmatrix}$$

9. $$A = \begin{array}{c|cc} & \text{ChCh} & \text{F} \\ \hline b & 3 & 2 \\ c & 1 & 2 \\ s & 2 & 1.5 \end{array}$$

$$B = \begin{array}{ccc} b & c & s \\ \hline 3 & 2.5 & 1 \end{array} \qquad C = \begin{array}{c|c} 5 & \text{Ch} \\ 6 & \text{F} \end{array}$$

BAC or $C^T A^T B^T$

10. x = \$ amount in gold
y = \$ " " money mkt
z = \$ " " in stocks

$x + y + z \le 100{,}000$

$.05x + .06y + .1z$ = Return to be maximized

$x + y \le .3(x + y + z) \rightarrow .7x + .7y - .3z \le 0$

$z \ge 3x \rightarrow -3x + z \ge 0$

11. Use rref see key

12. Only when the number of variables is equal to the number of nonzero rows
(on left) in the rref form.
There must be a unique solution and no redundant equations.
See key

13. see key

14. Leontief-Input-Output problem

$$\begin{array}{c|cc} & Ag & Oil \\ \hline Ag & .40 & .20 \\ Oil & .35 & .05 \end{array} = A \qquad D = \begin{bmatrix} 40 \\ 250 \end{bmatrix}$$

$$X = \begin{bmatrix} x \\ y \end{bmatrix} = \left(I_{(2)} - A\right)^{-1} D = \begin{array}{c} Ag \\ oil \end{array}\begin{bmatrix} 176 \\ 328 \end{bmatrix} \begin{array}{c} \text{million} \\ \$ \end{array}$$

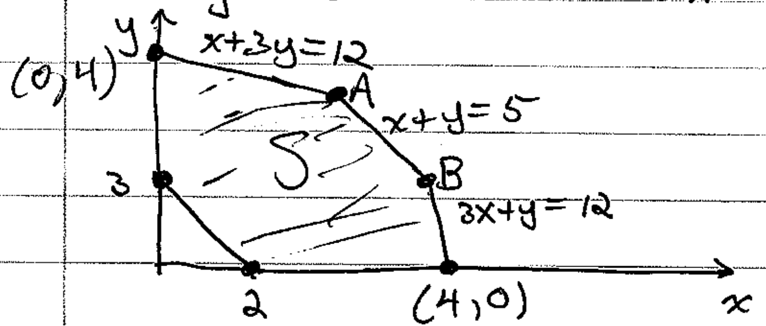
How much is used up:

$$X - D = \begin{array}{c} Ag \\ O \end{array}\begin{bmatrix} 136 \\ 78 \end{bmatrix} \text{million } \$$$

15. $3x + y \le 12$ $m = -3$

$x + 3y \le 12$ $m = -1/3$

$x + y \le 5$ $m = -1$ and $3x + 2y \ge 6$ $x, y \ge 0$



A: $x + 3y = 12$ and $x + y = 5$

$-(x + y = 5)$

$2y = 7$ $y = 7/2$ $x = \frac{3}{2}$ $A(\frac{3}{2}, \frac{7}{2})$

B: $x + y = 5$ and $3x + y = 12$

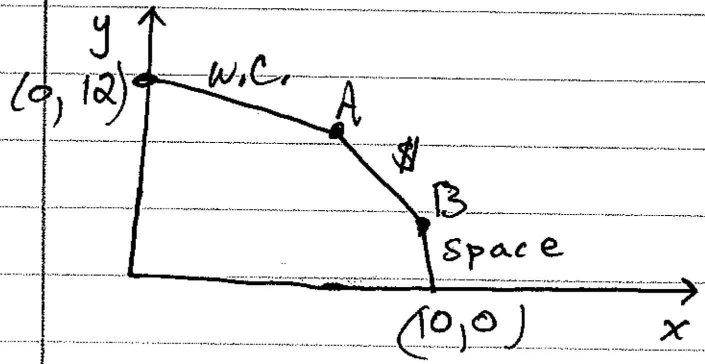
$2x = 7$ $x = \frac{7}{2}$ $y = 3/2$ $B(\frac{7}{2}, \frac{3}{2})$

Corners	$P = 4x + 5y$
$(0,3)$	15
$(0,4)$	20
$A(\frac{3}{2}, \frac{7}{2})$	$6 + \frac{35}{2} = 23.5$ ← max @ $(\frac{3}{2}, \frac{7}{2})$
$B(\frac{7}{2}, \frac{3}{2})$	$14 + \frac{15}{2} = 21.5$
$(4,0)$	16
$(2,0)$	8 ← min @ $(2,0)$

16. x = # puppies bought
y = # kittens bought

wkly = Care

	P, x	K, y	limit	slope
space	12	8	120	$-\frac{12}{8} = -1.5$
$	12	10	126	$-\frac{12}{10} = -1.2$
w.C.	3	3	36	-1



$$A = \text{rref}\begin{bmatrix} 3 & 3 & 36 \\ 12 & 10 & 126 \end{bmatrix} = \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 9 \end{array}\right] \; A(3,9)$$

$$B = \text{rref}\begin{bmatrix} 12 & 8 & 120 \\ 12 & 10 & 126 \end{bmatrix} = \left[\begin{array}{cc|c} 1 & 0 & 8 \\ 0 & 1 & 3 \end{array}\right] \; B(8,3)$$

corner	R = 80x + 70y
(0, 12)	840
A(3, 9)	240 + 630 = 870 ← max 3 puppies at 9 kittens
B(8, 3)	640 + 210 = 850
(10, 0)	800

A is not on the space line. 12(3) + 8(9) = 36 + 72 = 108 120 − 108 = 12 sq ft of extra space

20. a) P1 P2 P3 P4 P5 P6 P7 plays
prize1 prize2 prize3 prizes

Award prizes with no repeats.
$7P3$

b) Now repeats are allowed. Each prize can be awarded to any of the 7.
$7 \times 7 \times 7 = 7^3$

c) 1st find how many ways a play can get 2 prizes
$3C2$ ways to pair up two prizes
$(3C2)\, 7 \times 6$ ways to award the pair and the other prize.

At most 2 prizes go to any play:
$7P3 + (3C2)\, 7 \times 6$

Alternatively: $7^3 - 7$
= total − all 3 to one play

21. 12 spaces in the lineup. Choose 5 of the 12 for red, 4 of the 7 remaining for blue, the last 3 will be yellow.

$(12C5)(7C4)$

Missippi way: $\frac{12!}{5!4!3!}$

22. Similar to #21 $(70C1)(69C5)(64C10)$

Mississippi way $\frac{70!}{5!10!54!}$

(54! since 54 people get no prize)

23. 3 groups can be arranged in 3! ways.

$3!(5!3!4!)$

24. a) $1 - P(\text{no honeycrisp}) = 1 - \frac{33C5}{50C5}$

b) $1 - [P(\text{no honeycr}) + P(\text{exactly 1 honeycr})]$

$= 1 - \left[\frac{33C5}{50C5} + \frac{(17C1)(33C4)}{50C5}\right]$

c) $P(5 \text{ honeycr}) + P(5 \text{ gala}) + P(5 \text{ cameo})$

$= \frac{17C5 + 20C5 + 13C5}{50C5}$

d) $\frac{(17C2)33C3 + (20C3)(30C2) - (17C2)(20C3)}{50C5}$

25. Tosses of a die are independent. Each time we observe whether or not the top # is a (1 or 2) or (some other #).

The die is tossed 15 times

X is binomial, $n = 15$ $p = \frac{2}{5}$.

a) $P(X=6) = \text{binompdf}(15, 2/5, 6) \approx .206598$

b) $P(4 \leq X \leq 8) = \text{binomcdf}(15, 2/5, 8) - \text{binomcdf}(15, 2/5, 3) \approx .814451$

c) $\text{normalcdf}(3.5, 8.5, 15 \times .4, \sqrt{15 \times .4 \times .6})$

$\approx .812368$

26.

X	5	4	3	2	1	0
freq	55	40	25	15	5	10

total = 150

a) $P(X \geq 3) = \frac{25+40+55}{150} = \frac{12}{15} = 4/5$

b) $E(X) = 3.6\overline{3}$ using 1VAR STATS L_1, L_2

c) $\sigma X \approx 1.47158267$

d) $\text{med } X = 4$

e) mode $= 5$ most frequent

27. $P(E \cup F) = 1 - P((E \cup F)^c) = 1 - P(E^c \cap F^c)$

$= 1 - .06$

$= .94$

a) $P(E) + P(F) - x = .94$

$.7 + .8 - x = .94$ $\quad x = 1.5 - .94 = .56$

$x = P(E \cap F) = .56$

b) $.7 \times .8 = .56$ $\quad P(E \cap F)$ was found to be .56

yes, independent

28. a) Birthday type: $1 - \frac{12P6}{12^6}$

b) $\frac{12P1}{12^6} = \frac{12}{12^6} = \frac{1}{12^5}$

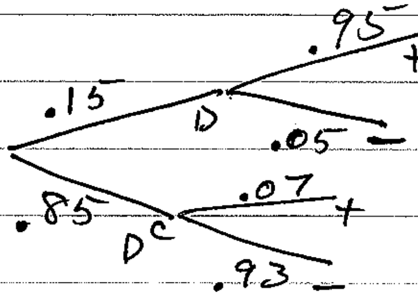
c) There are 6C3 ways to put 3 people in a group that gets 1 variety. Then we have $\underbrace{\text{group of 3}}$, $\underbrace{\text{1 other}}$ $\underbrace{\text{1 other}}$ $\underbrace{\text{1 other}}$

Assign 4 of the 12 varieties in 12P4 ways.

Multiplying, there are (6C3)(12P4) ways and dividing by the total ways to assign:

$(6C3)(12P4)/12^6$

29.



a) $P(+) = (.15)(.95) + (.85)(.07) = .2065$

b) $(.15)(.95) = .1425$ c) $.95$ read the tree

d) $\frac{.1425}{.2065} \cong .6901$

30. Let X be the number among the 10 who have the disease. X is binomial, $n = 10$, $p = .6901$.

a) $P(X \geq 5) = 1 - P(X < 5)$
$= 1 - \text{binomcdf}(10, .6901, 4)$
$\approx .05503$

b) $E(X) = np = 10 \times .6901 = 6.901$

31. $P(C) = \frac{100}{200} = .5 \quad P(M) = \frac{110}{200} = .55$

$P(C \cap M) = \frac{75}{200} = .375$

a) Does $P(C) \cdot P(M)$ equal $P(C \cap M)$?

$.5 \times .55 \neq .375$

dependent

b) $P(M/C) = \frac{75}{100} = .75 > P(M)$

yes, the coffee helped.

c) $P(M/C^c) = \frac{35}{100} = .35$

($35 = 110 - 75$, the # in the no coffee group who remembered)

32. a) 3 in favor, 4 against $P(win) = \frac{3}{7}$

b) $P(rain) = .4$

$P(\text{no rain}) = .6$

Odds in favor of rain are 4 : 6

33.

ckpt	1	2	3	
pass	.05	.06	.04	not detected
don't pass	.95	.94	.96	detected

a) $1 - P(\text{none detect}) = 1 - .05 \times .06 \times .04 = .99988$

b) $.95 + .94 - P(both) = .95 + .94 - .95 \times .94$ by independ.

$= .997$

34. $7.2(.4) + 6.9(.35) + 6.5(.25)$

$= 6.92$ lbs

35. Enter prices in L_1, #days in L_2

1-VAR STATS L_1, L_2

$\bar{x} = 50$

$\sigma_x \cong 2.64575$

36. $A = 4$ pts $B = 3$ pts $C = 2$ pts $D = 1$ pt $F = 0$

Let L_1 be 4 3 2 1 0

L_2 3 4 4 3 0

a) 1-VAR STATS L_1, L_2 $\bar{x} = 2.5$ = gpa for semester

b) Extend L_1 to include 3.0

L_2 to include 70

L_1 4 3 2 1 0 3.0

L_2 3 4 4 3 0 70

1-VAR STATS L_1, L_2 $\bar{x} = 2.91\overline{6}$

Alternatively, $\dfrac{2.5 \times 14 + 3.0 \times 70}{84} = 2.91\overline{6}$

37. a) normalcdf(80, 10^9, 75, 12) ≈ .33846
Rounded to 2 dec. places .34.

b) X is binomial, $n=10$ $p=.34$
$P(X \geq 3) = 1 - P(X<3)$
$= 1 -$ binomcdf(10, .34, 2)
$\cong .71623$

c) $E(X) = np = 3.4$

d) Now back to a random score, which is normally distributed:
normalcdf(70, 80, 75, 12) ≅ .3231

e) $P(S \leq a) = .7$
$a =$ invNorm(.7, 75, 12) ≈ 81.293
or a score of 81

f) $P(S \geq b) = .7$
$P(S < b) = .3$ $b =$ invNorm(.3, 75, 12)
≈ 68.707
or a score of 69.

38/40. X is binomial, $n=20$

$p = P(19 \leq L \leq 21)$

where L is the normally distributed length.

$p = \text{normalcdf}(19, 21, 20, .75)$
$\approx .8175775$ or $.82$

a) $E(X) = 20 \times .82 = 16.4$

b) $P(X \geq 14) = 1 - P(X < 14)$
$= 1 - \text{binomcdf}(20, .82, 13)$
$\cong .94633$

c) $P(15 \leq X \leq 18)$
$= \text{binomcdf}(20, .82, 18)$
$- \text{binomcdf}(20, .82, 14)$
$\approx .76257$

d) $L = \text{invNorm}(.25, 20, .75)$
$\cong 19.494$