2.4

A matrix is an array of numbers.

For example, $\begin{bmatrix} 1 & 3 & -1 & 2 \\ 4 & 6 & 3 & 0 \\ -2 & 1 & 5 & 7 \end{bmatrix} = A$.

This matrix has 3 rows and 4 columns, so it is 3×4.

The entry in row i column j is a_{ij}.

For example, $a_{23} = 3$ and $a_{14} = 2$.

We can add matrices of the same size by adding entries in the same position.

For example, $C = \begin{bmatrix} 2 & 0 \\ 1 & 5 \end{bmatrix}$ $D = \begin{bmatrix} 3 & -1 \\ 2 & 7 \end{bmatrix}$

then $C + D = \begin{bmatrix} 2+3 & 0-1 \\ 1+2 & 5+7 \end{bmatrix} = \begin{bmatrix} 5 & -1 \\ 3 & 12 \end{bmatrix}$

The calculator can also do this.

We can multiply a matrix by a number by multiplying each entry!

For example, $4\begin{bmatrix} 3 & -1 & 6 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 12 & -4 & 24 \\ 8 & 4 & 12 \end{bmatrix}$

The <u>transpose</u> of a matrix, called A^T for example, is the matrix formed by making row i of A be column i for each row of A.

Example: $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \end{bmatrix}$ then

$$A^T = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 3 & 2 \end{bmatrix}$$

Note that A is 2×3 and A^T is 3×2. The transpose can also be done in the calculator.

Practice Example: For $A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

and $B = \begin{bmatrix} 6 & 1 \\ -2 & 4 \\ 3 & -1 \end{bmatrix}$, work out

a) $3A^T + B$

b) $A + 5B^T$

Check in the calculator.

An example with meanings:

John and Mary invest in 3 accounts, gold, stocks and bonds.

John invests $8000 in gold, $10,000 in stocks, and $20,000 in bonds.

Mary invests $12,000 in gold, $20,000 in stocks, and $6000 in bonds.

a) Use a matrix to represent their investments. Label the rows and columns. Call this A.

b) How else could you represent the investments in a matrix?

c) If they each increase their investments by 25%, what number multiplied by A shows their investments?

d) After this 25% increase they add to their investments as follows:

John adds $1,000 in gold, $3,000 in stocks and $5,000 in bonds.
Mary adds $4,000 in gold, $8,000 in stocks and $1,000 in bonds.

Call the add-ons matrix B and write their total investment as a matrix.