3.2 Setting up Linear Programming Problems

What is a Linear Programming Problem?
Find the maximum or minimum value of a linear expression subject to certain linear constraints.
Ex. Maximize $P=2x+5y$ subject to $3x+4y\leq 10$, $5x+2y\leq 15$, $x,y\geq 0$.
P is the objective function and the inequalities are the constraints.

We start with word problems which we formulate. We will solve them later.

Ex. A manufacturing company produces two models of hibachis, model A and model B. Each model A requires 3 lbs of cast iron and 6 minutes of labor. Each model B requires 4 lbs of cast iron and 3 min. of labor. Profit per model A is \$2.00 and profit per model B is \$1.50. One thousand lbs of iron and 20 hours of labor are available. How many of each model should they produce to maximize profit.

Define variables according to what we are supposed to find: x= number of model A
y=number of model B
Set up a table with A,x and B,y heading the columns.

	A,x	B,y	limit
Iron	3	4	1000 lbs
Labor	6	3	1200 minutes
Profit	2	1.50	

The setup is: Maximize $P=2x+1.50y$ subject to $3x+4y\leq 1000$ and $6x+3y\leq 1200$, $x,y\geq 0$.

Ex. A finance company has \$20,000 for homeowner and auto loans. Homeowner loans have a 10% annual rate of return and auto loans have a 12% annual rate of return. The total amount in homeowner loans is to be more than or equal to 4 times the amount in . auto loans. How much of each type of loan should they extend to maximize return?
Define variables: x=amount in homeowner loans y=amount in auto loans
$x+y\leq 20000$, $x\geq 4y$, $x,y\geq 0$.

Example: This is a Standard Maximization Problem

A company produces two products, X and Y. Each unit of X requires an input of $20, 1.5 labor hours and 5 cubic feet of storage space. Each unit of Y requires an input of $30, 1 labor hour and 5 cubic feet of storage space. They have available $27000, 1200 labor hours and 5000 cubic feet of storage space. Profit per unit of X is $8 and profit per unit of Y is $10. How many of each should they produce to maximize profit and what is the maximum profit? How much of each resource, if any, is leftover?

Define variables:

x = the number of product X produced   y = the number of product Y produced

Set up a table for all the information:

inputs/unit	X	Y	limits
$=money	20	30	27000
L.H.=labor hours	1.5	1	1200
S.S.=storage space	5	5	5000
Profit/unit	8	10	

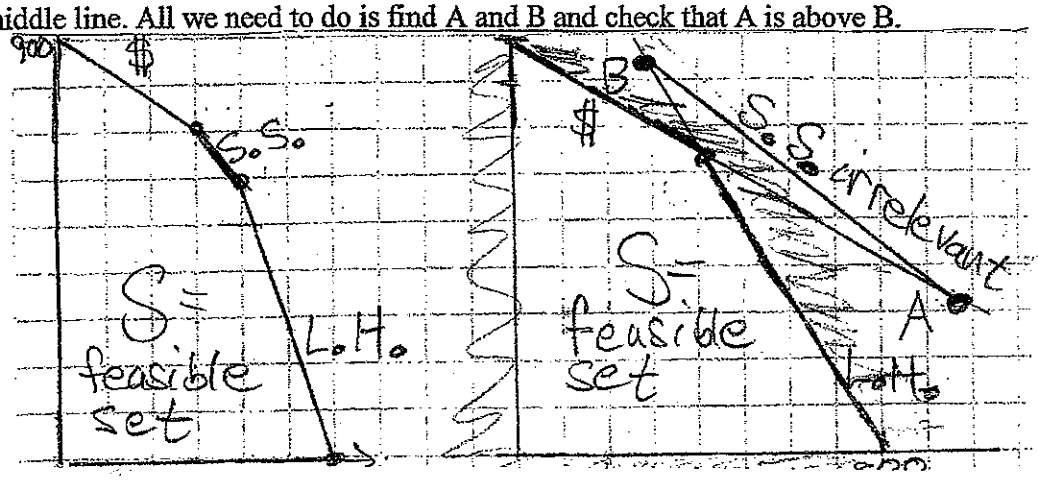
Write each constraint and make a table of the slopes and intercepts of the boundary lines:

	constraint	slope	y-int.	x-int.
$	$20x + 30y \leq 27000$	-2/3	900	1350
L.H.	$1.5x + y \leq 1200$	-1.5	1200	800
S.S.	$5x + 5y \leq 5000$	- 1	1000	1000

The flattest line has the lowest y-int. and is on the top.
The steepest line has the smallest x-int and is on the right at the bottom.
The middle line has the middle slope.

Sketch an approximate graph labeling the y-int of the top line and the x-int of the bottom line. Label the other two corners A and B with A the intersection of the flattest line, ($), and the middle line, (S.S); and B the intersection of the steepest line, (L.H.), and the middle line (S.S.)

How do we know it looks like the first picture and not the 2nd? In the 2nd picture, the intersection of the middle line and the bottom line is above the intersection of the top line and the middle line. All we need to do is find A and B and check that A is above B.



In the 2nd picture, the storage space is so plentiful that the S.S constraint is irrelevant.
Find A using rref on $ and S.S. or elimination of variables.
A(300,700)
Find B similarly using rref on S.S. and L.H. B(400, 600) We see that B is below A so the 1st picture is correct.

Make a table listing the corners and the profit at each:

corner	P=8x + 10y
(0, 900)	9000
A (300,700)	9400
B (400,600)	9200
(800,0)	6400

So A(300,700) is the most profitable. They should produce 300 of X and 700 of Y for a profit of $9400.

Final step, Find the leftovers if any.
Since A is below the L.H. line, the are leftover labor hours.
Plugging A into the left side of the L.H. constraint, we have
labor hours used = 1.5(300) + 700 = 1150 hours. There were 1200 available so we have 50 extra labor hours leftover.
Since A lies on each of the other constraints, there are no other leftover resources.

3.3 Graphical Solution of Linear Programming Problems

We assume the constraints describe a convex bounded set S. S does not cave in anywhere. We assume the constraints are all linear.

Consider P=ax+by for a and b both positive. As we increase P the graph of the line moves out away from the origin. If we increase P too much, no part of the line touches S.

The biggest value of P for which the line still touches S occurs at a corner point of S. The smallest value of P for which the line still touches S also occurs at a corner point of S.

Ex. Let S be defined by $x+y\le 6$, $2x+y\le 8$, $x,y\ge 0$ and $P=2x+3y$

Sketch S and graph P=6, P=12, and P=18. We see the largest value of P for which the line still touches S is 18 at x=0, y=6.