Math 142 Fall 2011 Exam 1 **Print** Name______________________
class time or section________________

There are 19 multiple-choice problems worth 4 points each followed by 3 work out problems worth 9 points each. Partial credit will be given only if work is shown.

"An Aggie does not lie, cheat, or steal or tolerate those who do"

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Your signature ____________________________

Key White test

Exam 1 Version B Math 142 Fall 2011

1. A manufacturer is able to sell 800 units of a certain product if the price per unit is \$50. For each decrease of \$3 in the price, he sells 60 more units. For p = price per unit and x = quantity sold, the demand equation is

C

$a)\ p=-20x+1800 \quad b)\ p=-0.05x+50 \quad c)\ p=-.05x+90$

$d)\ p=-0.0635x+50 \quad e)\ p=0.05x+10$

(x, p)
$(800, 50)$

The given point is $x=800$ $p=50$

slope: $\Delta p=-3$ if $\Delta x=60$ $\quad m=\frac{-3}{60}=-.05$

Pt. slope: $y-y_1=m(x-x_1)$

$y-50=-.05(x-800)=-.05x+40$

$p=y=-.05x+90$

2. A manufacturer of a certain product finds his daily fixed costs total \$110.40. Each unit costs an additional \$35.40. If he produces x units, all x units are sold at the demand price, p, which is $p=-0.2x+47$. The larger break even quantity is

d

$C(x)=35.40x+110.40$

$R(x)=-.2x^2+47x$

a) 66.32 b) 235 c) 29 d) 46 e) none of these

$P(x)=-.2x^2+11.60x-110.40$

Use calculator

3. A line $y=mx+b$ has a slope of 3.5. If x decreases by 4 then the change in y is

C

a) an increase of 14 b) an increase of 0.875 c) a decrease of 14

d) a decrease of 0.875 e) a decrease of 1.143

$\Delta y=m\Delta x$ for any line

$\Delta y=(3.5)(-4)=-14$ or a decrease of 14

$\Delta x=-4$

4. A substance decays at a rate proportional to the amount present. Initially it weighed 6g. After 50 years it weighed 2g. A formula for the weight in grams after t years is

$a)6(3^{-50t})$ $b)6(3^{t-50})$ $c)6(3^{-t/50)})$ $d)18^{-t/50}$ $e)18^{50-t}$

c

$$W(0)\left[\frac{W(t_1)}{W(0)}\right]^{t/t_1} = 6\left[\frac{2}{6}\right]^{t/50} = 6\left(\frac{1}{3}\right)^{t/50}$$

$$= 6\left(3^{-t/50}\right)$$

5. The function $y=f(x)$ can be described by: $f(0)=7$ and for each increase of 2 in x, the y value is **multiplied** by 25. The formula for $f(x)$ is

$f(x)$ is exponential. ab^x

$a)\ f(x)=9x+7$ $b)\ f(x)=7(5^x)$ $c)\ f(x)=7(25^x)$

b

$d)\ f(x)=12.5x+7$ $e)\ f(x)=35^x$

or $7(25^{x/2}) = 7(5^x)$

$f(0)=7$ $f(2)=7(25)$

$b^2 = 25$

$b = 5$

$a = f(0) = 7$

6. If $(e^x)^t e^y = B$ then it is true that

$a)\ xt+y=\ln B$ $b)\ txy=\ln B$ $c)\ xt+y=\log_B e$

a

$d)\ t+x+y=\ln B$ $e)\ txy=\log_B e$

$e^{xt}e^y = B$

$e^{xt+y} = B$ $\ln\left(e^{xt+y}\right) = \ln B$

$xt+y = \ln B$

7. If P dollars is invested at 4% annual interest rate compounded continuously, then the time it will take for the accumulated amount to triple is

$a)\ \dfrac{\ln 3 - \ln P}{.04}\ years \quad b)\ \dfrac{3}{\ln 4}\ years \quad c)\ \dfrac{\ln 3}{.04}\ years$

c

$d)\ \dfrac{\ln 3P}{4\ln P}\ years \quad e)\ \dfrac{\ln 3}{4}\ years$

Solve for t in $Pe^{.04t} = 3P$

$$Pe^{.04t} = 3P$$
$$e^{.04t} = 3$$
$$.04t = \ln 3$$
$$t = \frac{\ln 3}{.04}$$

8. B is a positive constant. Solve for x if $x>0$ and $4\ln\sqrt[4]{B-x} + \ln(B+x) = 4$.

a

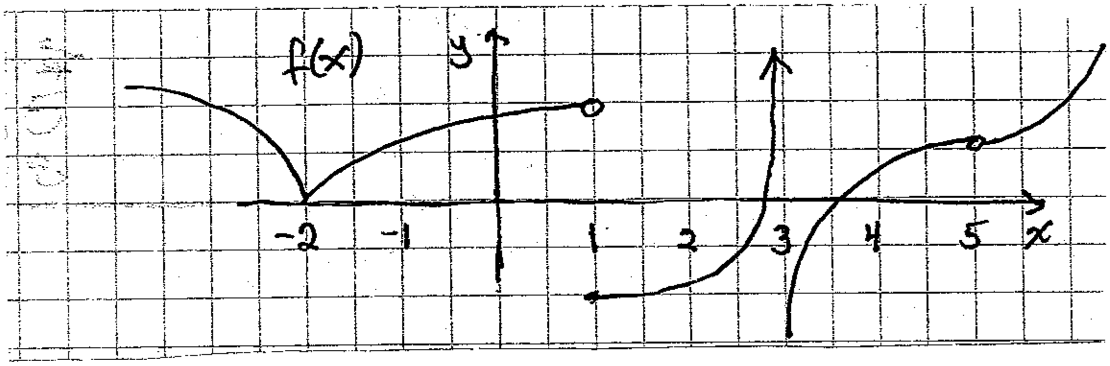
$a)\ x = \sqrt{B^2 - e^4} \quad b)\ x = \sqrt{B^2 - 4} \quad c)\ x = \sqrt{B^2 - \dfrac{4}{3}}$

$d)\ x = B^2 - \dfrac{4}{3}e \quad e)\ none\ of\ these$

$$4\ln(B-x)^{1/4} + \ln(B+x) = 4$$
$$\tfrac{1}{4}\cdot 4\ln(B-x) + \ln(B+x) = 4$$
$$\ln[(B-x)(B+x)] = 4$$
$$\ln[B^2 - x^2] = 4 \quad B^2 - x^2 = e^4$$
$$B^2 - e^4 = x^2$$

Given $x > 0$ so $x = \sqrt{B^2 - e^4}$

Shown is the graph for problems 9 and 10.



d

9. For what values of a does $\lim_{x \to a} f(x)$ not exist?

a) -2 and 1 b) -2, 1, 3, and 5 c) 1 and 5 d) 1 and 3 e) -2, 1, and 3

a

10. For what values of a is $f(x)$ not continuous at a?

a) 1, 3, and 5 b) -2, 1, 3, and 5 c) -2 and 1 d) 1 only e) none of these

a

11. $\lim_{x \to 5} \dfrac{3x^2 - 75}{x^2 - 8x + 15}$ is equal to

Substituting $x=5$ gives "$\frac{0}{0}$". Do algebra.

a) 15 b) 1 c) 0 d) 3 e) The limit does not exist.

$$\frac{3(x-5)(x+5)}{(x-5)(x-3)} = \frac{3(x+5)}{x-3} \xrightarrow[x \to 5]{\text{Plug in } 5} \frac{30}{2} = 15$$

$x \neq 5$ if $x \neq 5$

c

12. $\lim_{x \to \infty} \dfrac{5x^2 - 15x + 30}{2x^2 - 9x + 12}$ is equal to

a) ∞ b) 0 c) 2.5 d) $\frac{2}{3}$ e) none of these

$x \to \infty$ so look at the ratio of the leading terms. $\dfrac{5x^2}{2x^2} = \dfrac{5}{2} = 2.5$

13. $f(x) = \begin{cases} \dfrac{x+2}{x^2-9} & x<1 \\ 4x-8 & 1 \le x \end{cases}$

$\dfrac{x+2}{(x-3)(x+3)}$ has V. A. $x=-3$

Also $x=3$ is a V. A. but we use $4x-8$ near $x=3$, so $x=3$ is not a V. A. of $f(x)$.

$f(x)$ is not continuous at

d

a) -3 and 3 b) -3, 3 and 1 c) 1

d) -3 and 1 e) none of these

V. A. $x=-3$ check $x=1$: $\dfrac{1+2}{1-9} = \dfrac{-3}{8}$

$4(1)-8=-4$

$\lim_{x\to 1^-} f(x) = -3/8$ but

$\lim_{x\to 1^+} f(x) = -4$ gap at $x=1$

14. $g(x) = \begin{cases} \dfrac{4x-8}{x^2-4} & x<2 \\ 4x-7 & 2 \le x \le 5 \\ x & 5<x \end{cases}$

free due to typo

Fix typo

$g(x)$ is not continuous at

d) -2 and 5

Would have been -2 and 5

a) 2 only b) -2 and 2 c) -2 only d) 2 and 5 e) -2, 2 and 5

$g(x) = \begin{cases} \dfrac{4(x-2)}{(x-2)(x+2)} = \dfrac{4}{x+2} & x<2 \quad \text{V.A. } x=-2 \\ 4x-7 & 2 \le x \le 5 \\ x & 5<x \end{cases}$

Check $x=2$: $\lim_{x\to 2^-} g(x) = \dfrac{4}{2+2} = 1$

$\lim_{x\to 2^+} g(x) = 4(2)-7 = 1$

g(2) also is 1

g is continuous at $x=2$.

check $x=5$: $4(5)-7 \neq 5$ so g has a gap at 5

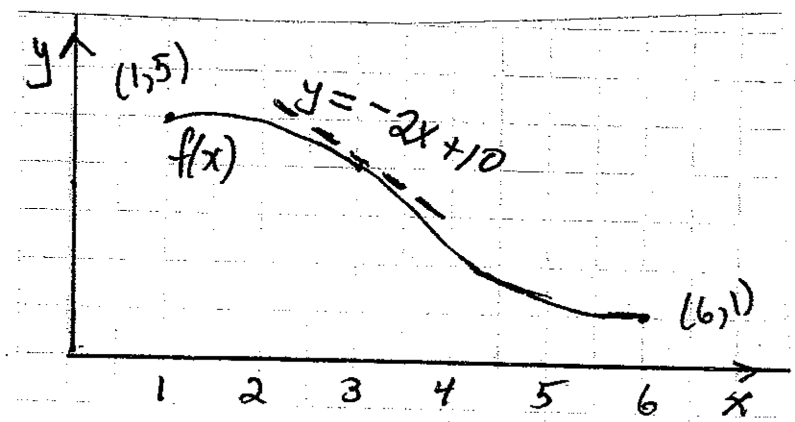
15. According to the limit definition of the derivative for $f(x)=\sqrt{x}$, $f'(x)$ is equal to

b

$a)\ \lim\limits_{x\to 0}\dfrac{\sqrt{(x+h)}-\sqrt{x}}{h}$ $\quad b)\ \lim\limits_{h\to 0}\dfrac{\sqrt{(x+h)}-\sqrt{x}}{h}$ $\quad c)\ \lim\limits_{x\to\infty}\dfrac{\sqrt{(x+h)}-\sqrt{x}}{h}$

$d)\ \lim\limits_{h\to 0}\dfrac{\sqrt{x}\ +h}{h}$ $\quad e)\ \lim\limits_{a\to 0}\dfrac{\sqrt{x}\ -\sqrt{a}}{a}$

Use the graph shown for 16 and 17. The line $y = -2x + 10$ is tangent to the graph at $x = 3$.



16. The average rate of change of *f(x)* over the interval [1, 6] is

a) -0.8 b) -2 c) -1.25 d) 1 e) cannot be found

a

slope from (1,5) to (6,1) is

$$\frac{1-5}{6-1} = \frac{-4}{5} = -.8$$

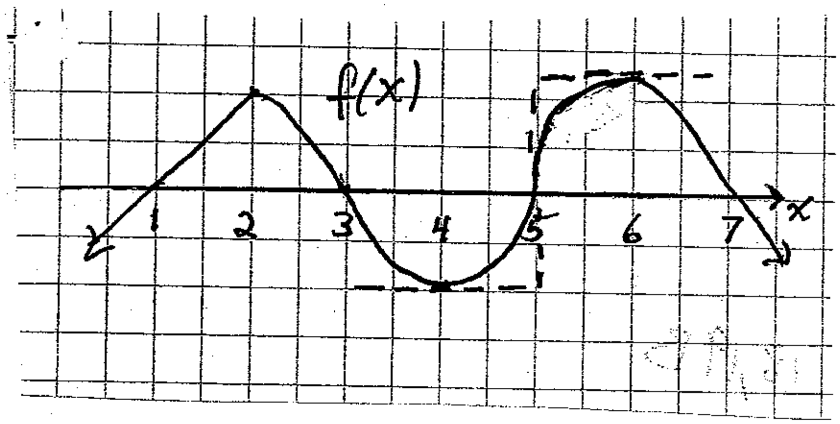
b

17. Refer to the graph above. The derivative of *f(x)* at $x = 3$, f '(3), is equal to

a) -0.8 b) -2 c) -1.25 d) 1 e) cannot be found

$f'(3) =$ "slope of $-2x+10$" $= -2$

Use the graph shown for 18 and 19.



18. At what x values is the derivative of $f(x)$ equal to 0? That is, $f'(x) = 0$ at the x values

a   a) 4 and 6   b) 2 only   c) 2 and 5   d) 1, 2, 3, 4 and 6

e) 1, 2, and 4   Horizontal tangents at 4 and 6

19. Refer to the graph above. $f'(x)$ does not exist at the x values

d   a) 4, 5, and 6   b) 2, 4, 5, and 6   c) 4 and 6   d) 2 and 5

e) $f'(x)$ exists for all x shown

sharp turn at $x = 2$.
Vertical tangent at $x = 5$.

Work out problems. Each problem is 9 points. Show all work.

1.

$$f(x)=\begin{cases} \dfrac{7x^2-63}{x^2-x-6} & x<3 \\ x & 3\le x<8 \\ 3x-16 & 8\le x \end{cases}$$

$\dfrac{7(x-3)(x+3)}{(x-3)(x+2)}$

a) Is f continuous at $x = -2$? Show why or why not using limits and the definition of continuity.

No. V. A. at $x=-2$.
$x+2$ does not cancel out of the denominator, and $f(-2)$ is undefined, even if it did.

b) $\lim_{x\to 3^-} f(x) = 8.4$ $\lim_{x\to 3^+} f(x) = 3$ Is f continuous at $x = 3$? No

$\parallel$

$\dfrac{7(3+3)}{3+2}$

$=\dfrac{42}{5} = 8.4$

x at $x=3$

gap

$\lim_{x\to 3} f(x)$ DNE

c) Is f continuous at $x = 8$? Show why or why not using limits and the definition of continuity.

$\lim_{x\to 8^-} x = 8$ $\lim_{x\to 8^+} (3x-16) = 3(8)-16 = 8$

$f(8) = 3(8)-16 = 8$ yes, continuous at $x=8$.

2. Find an equation of the tangent line to the graph of
$f(x) = 4x^3 + 6x^2 - 3x + 2$ at $x = 1$.

Find the point of tangency:

$f(1) = 4 + 6 - 3 + 2 = 9$ $\qquad$ $(1, 9)$

Find the slope: Find $f'(x)$ and plug in $x = 1$.

$f'(x) = 12x^2 + 12x - 3$

$f'(1) = 21$

Pt. Slope $\quad y - y_1 = m(x - x_1)$

$y - 9 = 21(x - 1)$

$y = 21(x - 1) + 9$

3. Find the derivative of each function.

a) $f(x) = 3(x+1)^7 + 6x^{-3} + 4$

shift rule and power rule

$f'(x) = 3 \cdot 7(x+1)^6 + 6 \cdot (-3) x^{-4} + 0$

$= 21(x+1)^6 - 18x^{-4}$

b) $g(x) = (x+2)\sqrt{x}$

Distribute so $g(x)$ is a sum of power functions.

$g(x) = x \cdot \sqrt{x} + 2\sqrt{x} = x^{3/2} + 2x^{1/2}$

$g'(x) = \frac{3}{2}x^{\frac{1}{2}} + x^{-1/2}$

c) $h(x) = \dfrac{x^3 - 4x^2 + 2}{x}$

Divide so $h(x)$ is a sum of power functions.

$h(x) = x^2 - 4x + 2x^{-1}$

$h'(x) = 2x - 4 - 2x^{-2}$