Math 142 Fall 2011 Exam 1 **Print** Name______________________

class time or section________________

There are 19 multiple-choice problems worth 4 points each followed by 3 work out problems worth 9 points each. Partial credit will be given only if work is shown.

"An Aggie does not lie, cheat, or steal or tolerate those who do"

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Your signature ____________________________

Key to yellow test

Exam 1 Version A   Math 142 Fall 2011

1. A manufacturer is able to sell 800 units of a certain product if the price per unit is \$50. For each decrease of \$3 in the price, he sells 75 more units. For p = price per unit and x = quantity sold, the demand equation is

$a)\ p=-0.04x+82 \quad b)\ p=-0.04x+18 \quad c)\ p=25x-450$

a

$d)\ p=-25x+2050 \quad e)\ p=0.065x+800$

The given point is $x=800$ if $p=50$ (x, p) $(800, 50)$

slope $= \frac{-3}{75} = -\frac{1}{25} = -.04$ since $\Delta p = -3$ if $\Delta x = 75$.

pt. slope $y - y_1 = m(x - x_1)$

$y - 50 = -.04(x - 800) = -.04x + 32$

$p = y = -.04x + 82$

2. A manufacturer of a certain product finds his daily fixed costs total \$78.40. Each unit costs an additional \$35.40. If he produces x units, all x units are sold at the demand price, p, which is $p=-0.2x+48$. $C(x) = 35.40x + 78.40$

The larger break even quantity is $R(x) = -.2x^2 + 48x$

a

a) 56   b) 240   c) 31.5   d) 63   e) none of these

$P(x) = -.2x^2 + 12.60x - 78.40$

Use calculator

3. A line $y = mx + b$ has a slope of 2.5. If x decreases by 6 then the change in y is

d

a) an increase of 15   b) an increase of 2.4   c) a decrease of 2.4

d) a decrease of 15   e) an increase of 8.5

$\Delta y = m\Delta x = 2.5(-6) = -15$ or a decrease of 15

4. A substance decays at a rate proportional to the amount present. Initially it weighed 3g. After 50 years it weighed 1.5g. A formula for the weight in grams after t years is

$a)\,3(2^{-t/50}) \quad b)\,4.5^{-50t} \quad c)\,3(2^{-50t}) \quad d)\,3(1.5^{50-t}) \quad e)\,4.5^{50-t}$

a

$W(0)\left[\frac{W(t_1)}{W(0)}\right]^{t/t_1} = 3\left[\frac{1.5}{3}\right]^{t/50} = 3\left[\frac{1}{2}\right]^{t/50}$

$= 3(2)^{-t/50}$

5. The function $y=f(x)$ can be described by: $f(0)=5$ and for each increase of 2 in x, the y value is **multiplied** by 9. The formula for $f(x)$ is

$f(x) = ab^x \quad a=5 \quad f(2)=45$

$a)\, f(x)=4.5x-0.5 \quad b)\, f(x)=5(9^x) \quad c)\, f(x)=5(3^x)$

$b^2=9$

c

$b=3$

$d)\, f(x)=4.5x+5 \quad e)\, f(x)=20x-15$

$f(x)=5(3^x)$

6. If $(10^y)^t 10^x = B$ then it is true that

$a)\; txy=\log B \quad b)\; ty+x=\log B \quad c)\; txy=\log_B 10$

b

$d)\; ty+x=\log_B 10 \quad e)\; t+y+x=\log\left(\frac{B}{10}\right)$

$10^{yt}\,10^x = B$

$10^{yt+x} = B$

$\log(10^{yt+x}) = \log B$

$yt+x = \log B$

7. If P dollars is invested at 6% annual interest rate compounded continuously, then the time it will take for the accumulated amount to triple is

$a)\ \dfrac{\ln 3-\ln P}{.06}\ years \quad b)\ \dfrac{3}{\ln 6}\ years \quad c)\ \dfrac{\ln 3}{.06}\ years$

c

$d)\ \dfrac{\ln 3P}{6\ln P}\ years \quad e)\ \dfrac{\ln 3}{6}\ years$

Solve for t in $Pe^{.06t} = 3P$

$$e^{.06t} = 3$$
$$.06t = \ln 3$$
$$t = \frac{\ln 3}{.06}$$

8. A is a positive constant. Solve for x if $x>0$ and $3\ln\sqrt[3]{A+x}+\ln(A-x)=2$.

$a)\ x=\sqrt{A^2-2} \quad b)\ x=\sqrt{A^2-e^2} \quad c)\ x=\sqrt{A^2-\dfrac{2}{3}}$

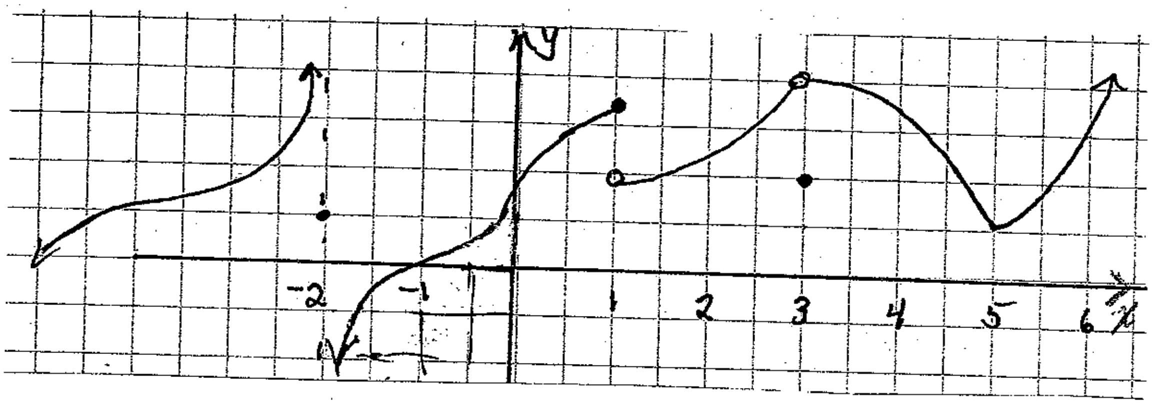
b

$d)\ x=A^2-\dfrac{2}{3}e \quad e)\ none\ of\ these$

$$3\ln(A+x)^{1/3}+\ln(A-x) = 2$$
$$3\cdot\tfrac{1}{3}\ln(A+x) + \ln(A-x) = 2$$
$$\ln[(A+x)\cdot(A-x)] = 2$$
$$\ln[A^2-x^2] = 2$$
$$A^2-x^2 = e^2$$
$$A^2-e^2 = x^2$$

Since $x>0$ was given
$x=\sqrt{A^2-e^2}$

Shown is the graph for problems 9 and 10.



a

9. For what values of a does $\lim_{x \to a} f(x)$ not exist?

a) -2 and 1 b) -2, 1, 3, and 5 c) 1 and 5 d) 1 and 3 e) -2, 1, and 3

b

10. For what values of a is $f(x)$ not continuous at a?

a) -2, 1, 3, and 5 b) -2, 1, and 3 c) -2 and 1 d) 1 only e) none of these

b

11. $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 - 7x + 12}$ is equal to

$$\frac{(x-3)(x+3)}{(x-3)(x-4)} \xrightarrow[x \to 3]{} \frac{3+3}{3-4} = -6$$

a) 1 b) -6 c) 0 d) The limit does not exist. e) none of these

d

12. $\lim_{x \to \infty} \dfrac{2x^2 - 15x + 30}{3x^2 - 9x + 12}$ is equal to

$a)\ 2.5$ $b)\ 0$ $c)\ \infty$ $d)\ \dfrac{2}{3}$ $e)$ *none of these*

Since $x \to \infty$, look at the ratio of the leading terms $\dfrac{2x^2}{3x^2} = 2/3$

13. $f(x) = \begin{cases} \dfrac{x+1}{x^2-4} & x<1 \\ 4x-8 & 1 \le x \end{cases}$

f(x) is not continuous at

C

a) -2, 2, and 1 b) -2 and 2 c) -2 and 1 d) 1 e) none of these

$\dfrac{x+1}{(x-2)(x+2)}$ has V.A. $x=-2$ and $-2<1$

At $x=1$: $\lim_{x\to 1^-} \dfrac{x+1}{x^2-4} = -\frac{2}{3}$ $\lim_{x\to 1^+}(4x-8) = -4$

gap at $x=1$

free due to typo on white test

14. $g(x) = \begin{cases} \dfrac{6x-18}{x^2-9} & x<3 \\ 4x-11 & 3 \le x \le 5 \\ x & 5<x \end{cases}$

$\dfrac{6(x-3)}{(x+3)(x-3)}$. V.A. $x=-3$

g(x) is not continuous at

e

a) -3 and 3 b) -3 only c) 3 only d) -3, 3 and 5 e) -3 and 5

$\lim_{x\to 3^-} g(x) = \dfrac{6}{3+3} = 1$ $\lim_{x\to 3^+} g(x) = 4(3)-11 = 1$

$g(3) = 4(3)-11 = 1$ so g is continuous at $x=3$

At $x=5$: $\lim_{x\to 5^-} g(x) = 4(5)-11 = 9$

$\lim_{x\to 5^+} g(x) = x\Big|_{x=5} = 5$

gap at $x=5$

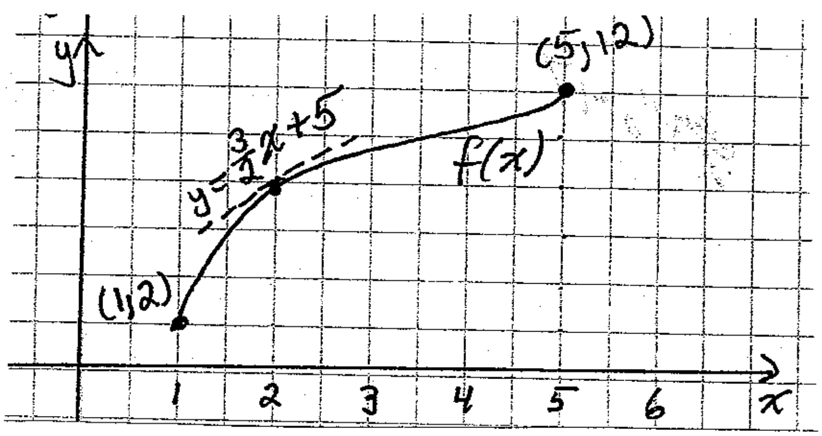
15. According to the limit definition of the derivative for $f(x)=x^3$, $f'(x)$ is equal to

$a)\ \lim_{x\to 0}\frac{(x+h)^3-x^3}{h}$ $b)\ \lim_{h\to 0}\frac{(x+h)^3-x^3}{h}$ $c)\ \lim_{x\to\infty}\frac{(x+h)^3-x^3}{h}$

b

$d)\ \lim_{h\to 0}\frac{x^3+h^3}{h}$ $e)\ \lim_{a\to 0}\frac{x^3-a^3}{a}$

Use the graph shown for 16 and 17. The line $y = 1.5x + 5$ is tangent to the graph at $x=2$.



16. The average rate of change of $f(x)$ over the interval [1, 5] is

a

a) 2.5 b) 12 c) 2.4 d) 2 e) cannot be found

slope from (1,2) to (5,12) is $\frac{12-2}{5-1}=\frac{10}{4}=2.5$

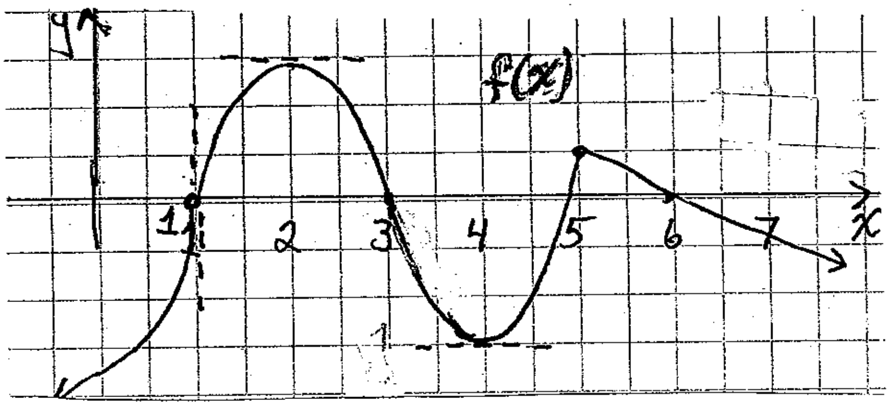
17. Refer to the graph above. The derivative of $f(x)$ at $x = 2$, $f'(2)$, is equal to

b

a) 8 b) 1.5 c) 5 d) 6 e) cannot be found

$f'(2)=$ "slope of $\frac{3}{2}x+5$" $=3/2=1.5$

Use the graph shown for 18 and 19.



d

18. At what x values is the derivative of $f(x)$ equal to 0? That is, $f'(x) = 0$ at the x values

a) 1, 2, 3 and 4   b) 1, 2 and 4   c) 1, 3, 4.8, and 6   d) 2 and 4

e) 1, 2, 4 and 5   horizontal tangents at $x=2$ and $x=4$

c

19. Refer to the graph above. $f'(x)$ does not exist at the x values

a) 1, 2, and 4   b) 2 and 4   c) 1 and 5   d) 1, 2, 4 and 5

e) $f'(x)$ exists for all x shown

vertical tangent at $x=1$
sharp turn at $x=5$

Work out problems. Each problem is 9 points. Show all work.

1.

$$f(x)=\begin{cases} \dfrac{5x^2-45}{x^2-x-6} & x<3 \\ x & 3\le x<7 \\ 3x-14 & 7\le x \end{cases}$$

$\dfrac{5(x-3)(x+3)}{(x-3)(x+2)}$ V.A. $x=-2$

a) $\lim_{x\to -2^-} f(x) = -\infty$ $\lim_{x\to -2+} f(x) = +\infty$ Is f continuous at $x = -2$? No V.A. $x=-2$

see calculator

b) Is f continuous at $x = 3$? Show why or why not using limits and the definition of continuity.

$\lim_{x\to 3^-} f(x) = \dfrac{5(3+3)}{3+2} = 6$ $\lim_{x\to 3^+} x = 3$

gap at $x=3$. f is Not continuous at $x=3$.

c) Is f continuous at $x = 7$? Show why or why not using limits and the definition of continuity.

$\lim_{x\to 7^-} f(x) = x\Big|_{x=7} = 7$ $\lim_{x\to 7^+} f(x) = 3(7)-14 = 7$

$f(7) = 3x-14\Big|_{x=7} = 7$ yes, f is continuous at $x=7$.

2. Find an equation of the tangent line to the graph of
$f(x) = 2x^3 - 5x^2 + 7x + 8$ at $x = 1$.

Pt. of tangency:

$f(1) = 2 - 5 + 7 + 8 = 12 \qquad (1, 12)$

Slope: Find $f'(x)$ and then $f'(1)$.

$f'(x) = 6x^2 - 10x + 7$

$f'(1) = 6 - 10 + 7 = 3 = \text{slope}$

Pt. Slope $\quad y - y_1 = m(x - x_1)$

$y - 12 = 3(x - 1)$

$$\boxed{y = 3(x-1) + 12}$$

3. Find the derivative of each function.

a) $f(x) = 4(x+1)^6 + 5x^{-2} + 9$ shift rule and powerrule.

$f'(x) = 4 \cdot 6(x+1)^5 + 5(-2x^{-3}) + 0$

$= 24(x+1)^5 - 10x^{-3}$

b) $g(x) = (x-1)\sqrt{x}$ Distribute $\sqrt{x}$ to write $g(x)$ as a sum of power functions.

$g(x) = x \cdot \sqrt{x} - \sqrt{x} = x^{3/2} - x^{1/2}$

$g'(x) = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}$

c) $h(x) = \dfrac{x^2+3x+4}{x}$ Divide to write $h(x)$ as a sum of power functions.

$h(x) = x + 3 + \frac{4}{x} = x + 3 + 4x^{-1}$

$h'(x) = 1 + 0 + 4(-x^{-2})$

$h'(x) = 1 - \frac{4}{x^2}$