Math 142 Exam 1 Review Solutions

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1. $$T(x) = \begin{cases} .04x & 0 \le x \le 30{,}000 \\ 1200 + .07(x - 30{,}000) = -900 + .07x & 30{,}000 < x \le 90{,}000 \\ 5400 + .15(x - 90{,}000) = -8100 + .15x & 90{,}000 < x \end{cases}$$

$T(30{,}000) = 1200$

$T(90{,}000) = 5400$

2. $$h(x) = \begin{cases} 2 + x & 0 \le x \le 5 \\ 7 + .8(x-5) = 3 + .8x & 5 < x \le 10 \end{cases}$$

At 5 yrs. $h = 7$
$h(5) = 7$

3. $C(x) = 30x + 375$ demandprice $p(x) = 70 - .25x$

$R(x) = 70x - .25x^2$

$P(x) = -.25x^2 + 40x - 375$

Break even when $x = 10$ and when $x = 150$

4. $A(t) = A(0)\, e^{-.012t}$

a) Solve $A(0)\, e^{-.012t} = \frac{1}{2} A(0)$ for t.

$$e^{-.012t} = \frac{1}{2}$$

$$-.012t = \ln \tfrac{1}{2}$$

$$t = \frac{-\ln \frac{1}{2}}{+.012} \approx 57.762 \text{ yrs}$$

≈ 57 yrs 9 months

b) $10 = A(0)$ so $A(t) = 10e^{-.012t}$

solve $2 = 10e^{-.012t}$ for t.

$$\frac{1}{5} = e^{-.012t}$$

$$\ln \tfrac{1}{5} = -.012t \qquad t = \frac{-\ln \frac{1}{5}}{.012} \approx 134.12 \text{yrs}$$

5. $Pe^{2r} = 10645.80 \quad Pe^{4r} = 11280.58$

$$\frac{11280.58}{10645.80} = \frac{Pe^{4r}}{Pe^{2r}} = e^{2r} \quad \ln(\text{—}) = 2r$$

$$\frac{1}{2}\ln\left[\frac{11280.58}{10645.80}\right] = r$$

$$\approx .0289586 \quad Pe^{2(.0289586)} = 10645.80$$

$P \approx \$10046.74$

6. $$W(t) = 2\left(\frac{2.5}{2}\right)^{t/3} = 2(1.25)^{t/3}$$

7. a) $$\log_3 \frac{3x+2}{x-2} = 4 \quad \frac{3x+2}{x-2} = 3^4 = 81$$

$$3x+2 = 81x - 162$$
$$164 = 78x \qquad x = \frac{164}{78} = \frac{82}{39}$$

b) $$\log_4(\sqrt{x+3})^2 = 2\cdot\log_4(x-3)^{-1} = 2 \quad \log_4(x-3)$$

$$\log_4(x+3) + \log_4(x-3) = 2$$

$$x^2 - 9 = 16 \qquad x^2 = 25$$

$\boxed{x=5}$ $\cancel{x=-3}$ $x=-5$

(makes arguments in log's negative)

8. $\ln 3 + 2\ln x + 2\ln(5 \quad) - \frac{1}{4}\ln(x^2+1)$

9. a) All 3 limits are 0.

b) $\lim_{x\to 0^-} f(x) = \infty \quad \lim_{x\to 0^+} f(x) = +\infty \quad \lim_{x\to 0} f(x)$ DNE

c) All 3 limits are -6.

d) $\lim_{x\to 2^-} f(x) = -\infty \quad \lim_{x\to 2^+} f(x) = +\infty \quad \lim_{x\to 2} f(x)$ DNE

e) All 3 limits are $8/3$.

10.

	x-value	graph	failure(s) in definition of continuity
a	-7	V.A.	$f(-7)$ is undefined and $\lim_{x\to -7} f(x)$ DNE
	-6	hole	$f(-6)$ is undefined so cannot equal $\lim_{x\to -6} f(x)$.
	0	V.A.	$\lim_{x\to 0} f(x)$ DNE
b	0	V.A.	$\lim_{x\to 0} f(x)$ DNE
	4	hole	$f(4)$ is undefined so $f(4) \neq \lim_{x\to 4} f(x)$.

11. $\lim_{x \to -\infty} f(x)$ $\qquad$ $\lim_{x \to +\infty} f(x)$

a) 8 $\qquad$ 8

b) 0 $\qquad$ 3

c) $-\infty$ $\qquad$ $+\infty$

d) 0 $\qquad$ 0

12. $$\lim_{h \to 0} \frac{3(x+h)^4 + 2\ln(x+h) - [3x^4 + 2\ln x]}{h}$$

13. i) $a = 9$ $\quad f(9) = 12\sqrt{9} = 36$ $\quad$ Pt. of tangency is $(9, 36)$.

$$f'(x) = 12\left(\tfrac{1}{2}x^{-1/2}\right) = \frac{6}{\sqrt{x}}$$

$$f'(9) = \frac{6}{\sqrt{9}} = 2$$ slope $= 2$.

Pt. Slope: $\boxed{y = 2(x-9) + 36}$

ii) $f(x) = \frac{1}{x^2} = x^{-2}$ $\quad a=1$

$f(1) = 1$ $\quad$ Pt of tangency is (1,1).

$f'(x) = -2x^{-3}$

$f'(1) = -2$ $\quad$ slope $= -2$

Pt. Slope $\quad y = -2(x-1) + 1$

iii) $f(x) = x(x+2)^2$ $\quad a=1$ $\quad f(1) = 9$

$= x(x^2 + 4x + 4)$ $\quad$ Pt. of tangency is (1, 9)

$f(x) = x^3 + 4x^2 + 4x$

Now we can use the power rule and the linear rule to find $f'(x)$.

$f'(x) = 3x^2 + 8x + 4$ $\quad f'(1) = 15$

$y = 15(x-1) + 9$

iv) $f(x) = x^2 - 4x + 9 \quad a = 2$

$f(2) = 4 - 8 + 9 = 5 \qquad (2, 5)$

$f'(x) = 2x - 4$

$f'(2) = 0 \qquad$ slope $= 0$

Line is $\boxed{y = 5}$

14. i) $y(1) = f(1) = 10$

Slope of $y = f'(1)$ so $f'(1) = 3$

ii) $f'(2)$ DNE since a vertical line's slope is undefined.

$f(2)$ could be any number so NEI

iii) $f(3) = 4 \qquad f'(3) = 0$

15. Check f is continuous.

$\lim_{x \to 1^-} (3x^{2/3} + 1) = 4 \quad \lim_{x \to 1^+} (x + 3) = 4$

and $f(1) = 4$ ✓

$\lim_{x \to 4^-} f(x) = 4 + 3 = 7 \quad \lim_{x \to 4^+} f(x) = 3\sqrt{4} + \frac{1}{4} \cdot 4 = 7$

$f(4) = 7$ ✓

None of the individual has any discontinuities.

Now use the power rule on each piece.

$$\left(3x^{2/3}+1\right)' = 3\cdot\tfrac{2}{3}x^{-1/3} = 2x^{-1/3} = \frac{2}{\sqrt[3]{x}}$$

$\boxed{x=0}$ is undefined at $x=0$. f has a vertical tangent at $x=0$.

$\boxed{x=1}$: Does the slope on the left at 1 match the slope ~~the~~ right at 1?

$$\left.\begin{array}{l} \text{left side slope} = \dfrac{2}{\sqrt[3]{1}} = 2 \\ \text{right side slope} = (x+3)' = 1 \end{array}\right\} \text{do not match}$$

f has a corner at $x=1$.

OK

$x=4$: The left side has slope 1.

$$\left.\left(3\sqrt{x}+\tfrac{1}{4}x\right)'\right|_{x=4} = \left.\frac{3}{2\sqrt{x}}+\frac{1}{4}\right|_{x=4} = \frac{3}{4}+\frac{1}{4} = 1$$

so the derivative at 4 exists and is equal to 1.

No derivative at $x=0$ or $x=1$.