Math 142 Exam 1 Review

1. An income tax schedule requires a person to pay 4% of the first $30,000 of income plus 7% of the income between $30,000 and $90,000 plus 15% of the income over $90,000. Write a piecewise definition of the tax, T, as a function of income x.

2. A tree seedling was planted when it was 2 ft. tall. It grew 1 ft. per year for the first 5 years and then grew a total of 4 ft. in the next 5 years. Write a piecewise definition of the height of the tree as a function of t = # of years since the planting.

3. In the production of a certain product, a company has fixed costs of $375 and each unit costs an additional $30 to produce. No one will buy the product until the price is below $70. For each decrease of $5 in the price 20 more units can be sold. Find the break even quantities.

4. A radioactive substance decays exponentially at the **continuous** rate of 1.2% per year.
a) Find the half life of the substance.
b) If initially the weight was 10 g, and now it weighs 2g, how many years old is it?

5. A principle, P, is invested at annual interest rate, r, compounded continuously. At 2 years the account grows to $10645.80. At 4 years it grows to $11280.58. Find the Principle rounded to the nearest cent and the interest rate rounded to 4 decimal places.

6. A culture begins with a weight of 2 g. It grows exponentially so that after 3 hours it weighs 2.5 g. Find the weight as a function of t = # of hours since it weighed 2g.

7. Solve for x:

a) $\log_3(3x+2) - \log_3(x-2) = 4$ b) $2\log_4\sqrt{x+3} = 2 + \log_4\left(\frac{1}{x-3}\right)$

8. Write as a sum/difference of simpler logarithms as much as possible:

$$\ln\left[\frac{3x^2(5x+2)^2}{\sqrt[4]{x^2+1}}\right]$$

9. $f(x) = \dfrac{x^3+4x^2-5x}{x^4-3x^3+2x^2}$ For each value of c below, evaluate

$\lim_{x\to c^-} f(x)$, $\lim_{x\to c^+} f(x)$, *and* $\lim_{x\to c} f(x)$ as a number, infinity, minus infinity, or state DNE.

a) c= -5 b) c=0 c) c=1 d) c=2 e) c=3

10. For each function, find all discontinuities and for each discontinuity tell i) what is seen in the graph and ii) all that fails in the definition of continuity (f is not defined, the value of f does not equal the limit, or the limit does not exist).

a] $f(x) = \begin{cases} \dfrac{x^2 - 36}{x^2 + 13x + 42} & x \le 0 \\ \ln x & 0 < x \end{cases}$ b]

$$f(x) = \begin{cases} \dfrac{2}{1 - e^x} & x < 0 \\ \dfrac{x^2 - 16}{x^2 - 2x - 8} & 0 \le x \end{cases}$$

11. For each function, find $\lim_{x \to \infty} f(x)$ *and* $\lim_{x \to -\infty} f(x)$ as a number, infinity, minus infinity or state DNE.

a) $f(x) = \dfrac{16x^3 - 24x + 7}{2x^3 + 18x^2 - 2000}$

b) $g(x) = \dfrac{3}{1 + 5e^{-x}}$

c) $h(x) = \dfrac{15x^2 + 1}{3x + 2}$

d) $k(x) = \dfrac{3x + 2}{15x^2 + 1}$

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12. State the limit definition of the derivative for $f(x) = 3x^4 + 2\ln x$

13. Find an equation of the tangent line at (*a*, *f(a)*) for the given function and the given value of *a*. Show all work but use the power rule and linear rule, not the limit definition, to find the slope.

i) $f(x) = 12\sqrt{x}$ $a = 9$ *ii*) $f(x) = \dfrac{1}{x^2}$ $a = 1$ *iii*) $f(x) = x(x+2)^2$ $a = 1$

iv) $f(x) = x^2 - 4x + 9$ $a = 2$

14. The tangent line to f(x) at a is given for several values of a. At each a value, find f(a) and f '(a), or state DNE or state NEI.

i) The tangent line at a = 1 is y = 3x + 7.

ii) The tangent line at a = 2 is x = 2.

iii) The tangent line at a = 3 is y = 4.

15. Find all values of a at which f '(a) does not exist (f is not differentiable at a). Check that f is continuous first.

$$f(x) = \begin{cases} 3x^{\frac{2}{3}} + 1 & x < 1 \\ x + 3 & 1 \le x < 4 \\ 3\sqrt{x} + \frac{1}{4}x & 4 \le x \end{cases}$$

16. Graph the given function in the calculator window Xmin=-5, Xmax=5, Ymin =-5 Ymax=5 and determine where the function is not differentiable.

$a)$ $f(x) = (x^2 - 4)^{\frac{2}{3}}$ $b)$ $f(x) = (x^2 - 9)^{\frac{1}{3}}$

$c)$ $f(x) = |x^4 - 9x^2|$ For c, change the window so Ymin=0 and Ymax=100