Yellow Exam 2

Math 142 Fall 2011 Exam 2 **Print** Name Key

class time or section ________

There are 16 multiple-choice problems worth 4 points each followed by 3 work out problems worth 12 points each. Partial credit will be given only if work is shown.

"An Aggie does not lie, cheat, or steal or tolerate those who do"

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Your signature ________________

Math 142 Exam 2 B Fall 2011 Print Name ______________________

Class time or section number ____________

For problems 1 and 2, the profit function is given by

$P(x) = -0.03x^2 + 15x - 1000$ dollars for $x$ units produced.

1. According to the marginal profit approximation, the profit at production level 320 units is

a) decreasing by about \$0.06 per additional unit

b) decreasing by about \$4.20 per additional unit.

c) increasing by about \$4.20 per additional unit.

d) decreasing by about \$4.23 per additional unit.

e) increasing by about \$4.23 per additional unit.

$P'(x) = -0.06X + 15$

$P'(320) = -4.20$

2. The average profit per unit at production level 320 is

a) decreasing by about by about 0.013125 per unit per unit.

b) increasing by about 5.0875 per unit per unit.

c) decreasing by about 0.029 per unit per unit.

d) increasing by about 0.029 per unit per unit.

e) none of these

$AP = \frac{P(x)}{X} = -.03x + 15 - \frac{1000}{x}$

$\left(\frac{P(x)}{X}\right)' = -.03 + \frac{1000}{x^2}$

$\left.\left(\frac{P(x)}{x}\right)'\right|_{x=320} = -.03 + \frac{1000}{320^2}$

$= -.0202\ldots$

Find the derivative of each function in problems 3 through 7.

3. $f(x)=(x^2+4)e^{7x}$. $f'(x)=$ $2xe^{7x}+7(x^2+4)e^{7x}$

$= (7x^2+2x+28)e^{7x}$

$a)\quad 14xe^{7x}$ $\qquad b)\quad (x^2+4)7xe^{7x-1}+2xe^{7x}$

$c)\quad 14x^2e^{7x-1}$ $\qquad d)\quad e^{7x}(7x^2+2x+28)$

$e)\quad e^{7x}(x^2+2x+4)$

4. $f(x)=\dfrac{x-3}{x^2+9}$ $\qquad f'(x)=$

$a)\quad \dfrac{1}{2x}$ $\qquad b)\quad \dfrac{-1}{(x+3)^2}$ $\qquad c)\quad \dfrac{x^2-6x-9}{(x^2+9)^2}$

$d)\quad (x^2+9)^{-1}-(x-3)(x^2+9)^{-2}$ $\qquad e)\quad \dfrac{6x+9-x^2}{(x^2+9)^2}$

$$f'(x)=\frac{x^2+9-(x-3)\cdot 2x}{(x^2+9)^2}$$

$$=\frac{x^2+9-2x^2+6x}{(x^2+9)^2}=\frac{9+6x-x^2}{(x^2+9)^2}$$

5. $f(x) = e^{x^3+6x}$ $\qquad f'(x) =$

$x^3+6x$

a) $(3x^2+6)e^{x3+6x}$ $\qquad$ b) $(x^3+6x)e^{x^3+6x-1}$ $\qquad$ c) $6xe^{3x^2+6}$

d) $(3x^2+6)e^{x^3+6x-1}$ $\qquad$ e) $e^{3x^2+6}$

$f'(x) = (3x^2+6)e^{x^3+6x}$

It was supposed to be (a) but for the typo.

6. $f(x) = \dfrac{x}{e^x+1}$ $\qquad f'(x) = \dfrac{1\cdot(e^x+1) - x\cdot e^x}{(e^x+1)^2}$

a) $\dfrac{e^x+1-xe^x}{(e^x+1)^2}$ $\qquad$ b) $\dfrac{xe^x-e^x-1}{(e^x+1)^2}$ $\qquad$ c) $\dfrac{1}{e^x}$

d) $\dfrac{e^x+1-xe^x}{e^{2x}+1}$ $\qquad$ e) $\dfrac{1-x}{4}$

7. $f(x)=(3x-1)^2(2x+1)^3 \qquad f'(x)=$

$a)\ \ 36(3x-1)(2x+1)^2 \qquad b)\ \ (3x-1)(2x+1)^2(13x-1)$

$c)\ \ 30x(3x-1)(2x+1)^2 \qquad d)\ \ (3x-1)(2x+1)^2$

$e)\ \ (3x-1)(2x+1)^2(10x+5)$

$$f'(x)=2(3x-1)\cdot 3(2x+1)^3+(3x-1)^2\cdot 3(2x+1)^2\cdot 2$$

$$=6(3x-1)(2x+1)^2[\underbrace{2x+1+3x-1}_{5x}]$$

$$=30x(3x-1)(2x+1)^2$$

8. A cost function is given by $C(x)=0.4x^2+25x+150$.
The **marginal average cost** function is

$a)\ \ 0.4x+25+150x^{-1} \qquad b)\ \ 0.8x+25$

$c)\ \ \dfrac{0.8x+25}{x} \qquad d)\ \ 0.4-\dfrac{150}{x^2} \qquad e)\ \ \dfrac{-0.4x^2+150}{x^2}$

$$AC=\frac{C(x)}{x}=0.4x+25+\frac{150}{x}$$

$$MAC=\left(\frac{C(x)}{x}\right)'=0.4-\frac{150}{x^2}$$

Problems 9 and 10, are about the function, *f(x)*. The derivative of *f(x)* is given by

$$f'(x) = (x-2)(x+1)^2.$$

9. Which is/are the x value(s) of the relative extrema of the function, *f(x)*?

a) max at x= 2 and no min.

b) max at x= -1 and min at x= 2.

c) min at x= -1 and max at x=2

d) max at x= -1 and min at x= 1.

e) min at x= 2 and no max.

sign $f'$: $-$ | $-1$ | $-$ | $2$ | $+$ → $x$

$f$: ↓ ↓ ↗

relative min @ x=2.
no max.

10. The function, *f(x)*, has inflection point(s) at

a) x = -1 only.

b) x = -1 and x= 1.

c) x = 1 only.

d) no inflection points

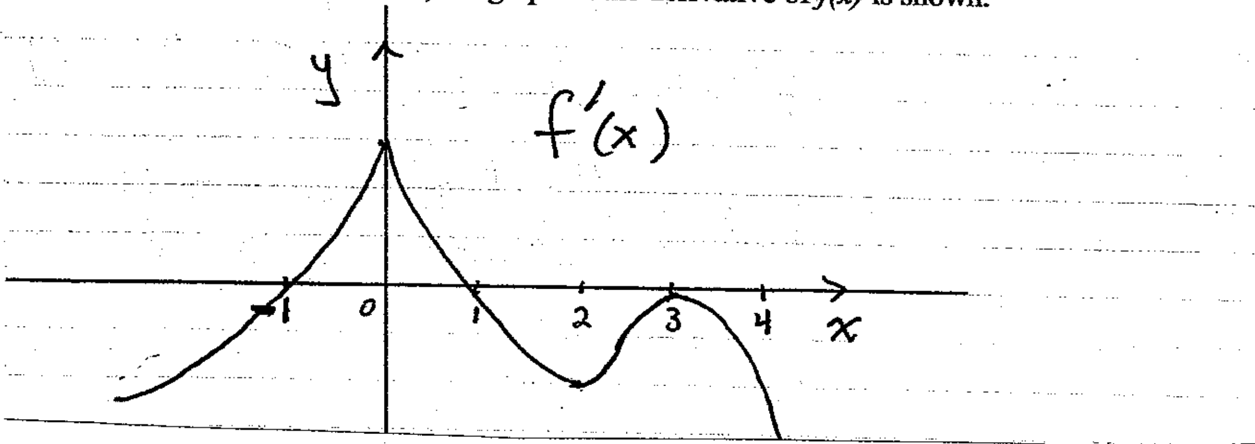
e) x = 0 only

$$f''(x) = (x+1)^2 + 2(x-2)(x+1)$$
$$= (x+1)[x+1+2x-4]$$
$$= (x+1)(3x-3)$$

$f''$: $+$ | $-1$ | $-$ | $1$ | $+$ →

$f$: ∪ IP ∩ IP ∪

For problems 11 and 12, the graph of the derivative of *f(x)* is shown.




11. The function with the derivative shown has relative extrema

a) max at x= 0 and x=3, min at x=2.

b) max at x = -1 and min at x=1.

c) min at x = -1 and max at x = 1.

d) max at x = 3 only.

e) none of these.

sign $f'$: $-$ | $-1$ | $+$ | $1$ | $-$ | $3$ | $-$

$f$: ↓ ↗ ↓ ↓

relative min @ $x = -1$

rel. max @ $x=1$

12. *f(x)* has inflection point(s) at

a) x= 0, x= 2, and x= 3.

b) x= -1 ,x = 1 and x = 3.

c) x = 2 and x = 3

d) about x= 2.5

e) none of these.

$f'$: ↗ | $0$ | ↘ | $2$ | ↗ | $3$ | ↘

sign $f''$: $+$ $-$ $+$ $-$

$f$: ∪ ∩ ∪ ∩

IP $x=0$

IP $x=2$

IP $x=3$

13. The derivative of $f(x) = \ln\left[(x^4 + e^x)(x-2)^5\right]$ is

$f'(x) =$

$f(x) = \ln(x^4+e^x) + 5\ln(x-2)$

a) $\dfrac{4x^3 + e^x}{x^4 + e^x} + \dfrac{5}{x-2}$

$f'(x) = \dfrac{4x^3+e^x}{x^4+e^x} + \dfrac{5}{x-2}$

b) $\dfrac{4}{x} + \dfrac{20}{x-2}$

c) $\dfrac{4}{x} + \dfrac{5}{x-2}$

d) $\dfrac{4}{x} + 1 + \dfrac{5}{x-2}$

e) *none of these*

14. The table shows the values of f ' and f " at three x-values.

| x | 0 | 1 | 2 |
|---|---|---|---|
| f'(x) | 0 | 1 | 0 |
| f"(x) | -2 | 1 | 3 |

According to the 2nd derivative test, f(x) has

The test only applies where $f'(a) = 0$, so only at $x=0$, $x=2$.

a) a relative max at x=0 and relative min at x=1 and x=2.

$f''(0) < 0$ rel max @ x=0

b) a relative min at x=1 and the test fails at x=0 and x=2.

c) a relative max at x=0 and relative min at x=2.

$f''(2) > 0$ rel. min @ x=2

d) a relative max at x=1.

e) a relative min at x=0 and relative max at x = 1 and at x=2.

15. For a certain product the demand quantity at price p dollars is $f(p) = e^{-.18p^2}$. Demand is inelastic for

a) $0 < p < 1.67$ b) $p > 1.67$ c) $0 < p < 2.78$ d) $p > 2.78$ e) $0 < p < 5.55$

$f'(p) = -.36pe^{-.18p^2}$

$E(p) = \dfrac{-p(-.36p\cancel{e^{-.18p^2}})}{\cancel{e^{-.18p^2}}} = .36p^2$

$E(p) = 1$ if $.36p^2 = 1$ $p^2 = \dfrac{100}{36}$ $p = \dfrac{10}{6} = 1.67$

16. The elasticity function for a product is $E(p) = \dfrac{p^2}{144 + p^2}$.

If the price increases from \$16 to \$17 then the approximate per-cent change in demand is

a) 3.84% and revenue will decrease.

b) 2.744% and revenue will decrease.

c) 2.744% and revenue will increase.

d) 4% and revenue will decrease.

e) 4% and revenue will increase.

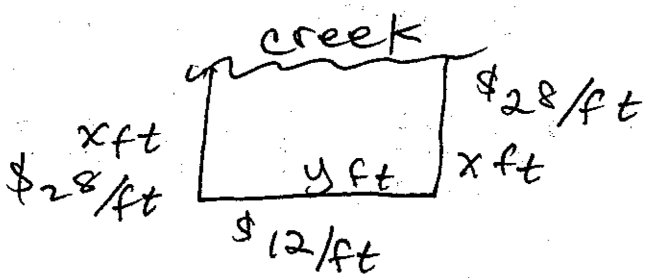
$E(p) < 1$ for all p so revenue in increasing. since $E(16) < 1$.

$E(16) \cdot \left(\dfrac{17-16}{16}\right) \times 100\%$

$= \dfrac{16^2}{400} \cdot \dfrac{1}{16} \times 100\% = 4\%$

There are three work out problems. Each problem is 12 points.
**Show all work on each problem.**

1. A rectangular area of 4,200 square feet is to be bordered by a creek on one side and fencing on the other 3 sides. The material for the side opposite the creek costs \$12 per foot. The material for the other two sides costs \$28 per foot. Find the dimensions that will minimize the cost. **After you find your answer, use the 2nd derivative test to show this gives the minimum cost.**




$A = 4200$ sq. ft.

$xy = 4200$

$\text{Cost} = 28x + 28x + 12y = 56x + 12y$

$y = \dfrac{4200}{x}$

$C(x) = 56x + 12\left(\dfrac{4200}{x}\right)$ to be minimized.

solve $C'(x) = 0$

$C'(x) = 56 - \dfrac{12(4200)}{x^2} = 0$ if $56 - 12\left(\dfrac{4200}{x^2}\right) = 0$

$56 = \dfrac{12(4200)}{x^2}$ ; $56 \cdot x^2 = 12(4200)$

$x^2 = \dfrac{12(4200)}{56}$

$x = \sqrt{\dfrac{12(4200)}{56}} = 30$

$y = \dfrac{4200}{30} = 140$

2nd Deriv. test

$C''(x) = \dfrac{2(12)(4200)}{x^3} > 0$ at $x = 30$ min

$x=30$

2. a) The derivative of a function, $f(x)$, is given by

$$f'(x) = (x+1)^2(x-2)(x-3)$$

Make a sign chart for $f'(x)$.

4 pts

$+ \quad + \quad - \quad +$

$-1 \quad 2 \quad 3 \quad x$

$\nearrow \quad \nearrow \quad \searrow \quad \nearrow$

Answer with all possible $x$-values or state none exists.

$f(x)$ has a min at $x = 3$ and a max at $x = 2$

$f(x)$ is increasing on $(-\infty, 2)$ and $(3, \infty)$

8 pts

b) The derivative of $g(x)$ is given by $g'(x) = x^2 e^{-x^3/12}$.

$g(x)$ has inflection point(s) at $x = 0$ and $x = 2$

$g(x)$ is concave up on $(0, 2)$.

$$g''(x) = 2x\, e^{-\frac{x^3}{12}} + x^2\left(-\frac{3x^2}{12} e^{-\frac{x^3}{12}}\right)$$

$$= e^{-\frac{x^3}{12}}\left(2x - \frac{1}{4}x^4\right)$$

$e^{-x^3/12} > 0$ for all $x$.

$2x - \frac{1}{4}x^4 = x\left(2 - \frac{1}{4}x^3\right) = 0$ at $x = 0$ and $x = 2$

$g''$: $- \quad + \quad -$

$0 \quad 2 \quad x$

$g$: $\frown \quad \smile \quad \frown$

test $x = 1$, $1(2 - \frac{1}{4}) > 0$

test $x = -1$ $(-1)(2 + \frac{1}{4}) < 0$

test $x = 3$, $< 0$

3. Find the derivative of each function. Do not simplify the derivative.

a) $f(x) = \log\left[\dfrac{(x^5+2)^3}{e^{x^2}+1}\right] = 3\log(x^5+2) - \log(e^{x^2}+1)$

$$f'(x) = \left(\frac{15x^4}{x^5+2} - \frac{2xe^{x^2}}{e^{x^2}+1}\right)\frac{1}{\ln 10}$$

b) $g(x) = \dfrac{e^{5x}+1}{(x^2+4)^2}$

$$g'(x) = \frac{5e^{5x}(x^2+4)^2 - (e^{5x}+1)(2)(x^2+4)(2x)}{(x^2+4)^4}$$