

White Exam

Math 142 Fall 2011 Exam 2 **Print Name** Key
class time or section _____

There are 16 multiple-choice problems worth 4 points each followed by 3 work out problems worth 12 points each. Partial credit will be given only if work is shown.

"An Aggie does not lie, cheat, or steal or tolerate those who do"

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Your signature _____

For problems 1 and 2, a profit function is given by

$$P(x) = -0.04x^2 + 20x - 1000 \text{ dollars for } x \text{ units produced.}$$

1. According to the marginal profit approximation, the profit at production level 210 units is

a) decreasing by about \$0.08 per additional unit.

$$P'(x) = -0.08x + 20$$

b) decreasing by about \$3.20 per additional unit.

$$P'(210) = -0.08(210) + 20$$

c) increasing by about \$3.20 per additional unit.

$$= 3.20$$

d) decreasing by about \$3.16 per additional unit.

e) increasing by about \$3.16 per additional unit.

2. The average profit per unit at production level 210 is

$$AP = \frac{P(x)}{x}$$

a) decreasing by about 0.0152 per unit per unit.

$$= -0.04x + 20 - \frac{1000}{x}$$

b) increasing by about 0.0152 per unit per unit.

c) decreasing by about by about .0377 per unit per unit.

$$(AP)'(x) = -0.04 + \frac{1000}{x^2}$$

d) increasing by about .0377 per unit per unit.

$$(AP)'(210) = -0.0173, \dots$$

e) none of these

Find the derivative of each function in problems 3 through 7.

3. $f(x) = (x^2 + 1)e^{6x}$ $f'(x) =$

a) $12xe^{6x}$ b) $(x^2 + 1)6xe^{6x-1} + 2xe^{6x}$

c) $12x^2e^{6x-1}$ d) $e^{6x}(x^2 + 2x + 1)$

e) $e^{6x}(6x^2 + 2x + 6)$

$$f'(x) = 2xe^{6x} + 6(x^2 + 1)e^{6x}$$

$$= (6x^2 + 2x + 6)e^{6x}$$

4. $f(x) = \frac{x-2}{x^2+4}$ $f'(x) =$

a) $\frac{4+4x-x^2}{(x^2+4)^2}$ b) $\frac{-1}{(x+2)^2}$ c) $\frac{x^2-4x-4}{(x^2+4)^2}$

d) $\frac{1}{2x}$ e) $(x^2+4)^{-1} - (x-2)(x^2+4)^{-2}$

$$\frac{x^2+4 - 2x(x-2)}{(x^2+4)^2} = \frac{x^2+4 - 2x^2+4x}{(x^2+4)^2}$$

$$= \frac{4+4x-x^2}{(x^2+4)^2}$$

$$5. f(x) = e^{x^2-5x} \quad f'(x) = (2x-5)e^{x^2-5x}$$

$$a) (x^2-5x)e^{x^2-5x-1} \quad b) e^{2x-5} \quad c) 2e^{2x-5}$$

$$d) (2x-5)e^{x^2-5x} \quad e) (2x-5)e^{x^2-5x-1}$$

$$6. f(x) = (3x-1)^2(2x+1)^3 \quad f'(x) =$$

$$a) 30x(3x-1)(2x+1)^2 \quad b) (3x-1)(2x+1)^2(13x-1)$$

$$c) 36(3x-1)(2x+1)^2 \quad d) (3x-1)(2x+1)^2$$

$$e) (3x-1)(2x+1)^2(10x+5)$$

$$\begin{aligned} f'(x) &= 2(3x-1)(2x+1)^3 \cdot 3 + (3x-1)^2 \cdot 3(2x+1)^2 \cdot 2 \\ &= 6(3x-1)(2x+1)^2 [2x+1 + 3x-1] \\ &= 6(3x-1)(2x+1)^2 (5x) \\ &= 30x(3x-1)(2x+1)^2 \end{aligned}$$

$$7. f(x) = \frac{x}{e^x + 1} \quad f'(x) =$$

$$a) \frac{xe^x + x - e^x}{(e^x + 1)^2} \quad \textcircled{b) \frac{e^x + 1 - xe^x}{(e^x + 1)^2}} \quad c) \frac{1}{e^x}$$

$$d) \frac{e^x + 1 - xe^x}{e^{2x} + 1} \quad e) \frac{1-x}{4}$$

$$f'(x) = \frac{e^x + 1 - xe^x}{(e^x + 1)^2}$$

8. A cost function is given by $C(x) = 0.4x^2 + 25x + 150$.
The marginal average cost function is

$$a) 0.4x + 25 + 150x^{-1} \quad b) 0.8x + 25$$

$$c) \frac{0.8x + 25}{x} \quad \textcircled{d) 0.4 - \frac{150}{x^2}} \quad e) \frac{-0.4x^2 + 150}{x^2}$$

$$AC = \frac{C(x)}{x} = 0.4x + 25 + 150x^{-1}$$

$$MAC = \left(\frac{C(x)}{x} \right)' = 0.4 - \frac{150}{x^2}$$

Problems 9 and 10, are about the function, $f(x)$. The derivative of $f(x)$ is given by $f'(x) = (x-1)(x+2)^2$.

9. Which are the relative extrema of the function, $f(x)$?

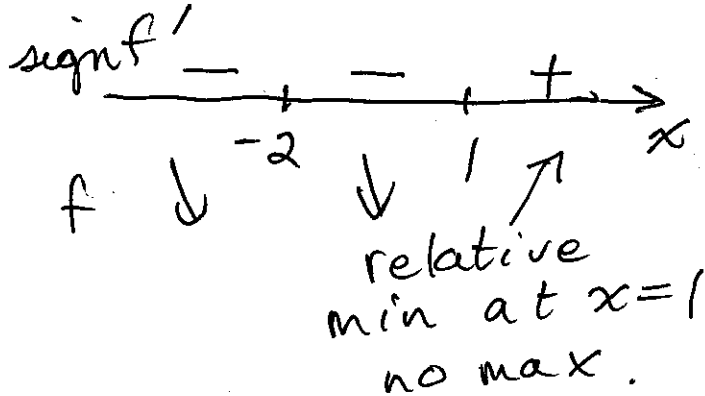
a) max at $x = -2$ and min at $x = 1$.

b) max at $x = 1$ and no min.

c) max at $x = -2$ and min at $x = 0$

d) max at $x = 1$ and min at $x = -2$.

e) min at $x = 1$ and no max.



10. The function, $f(x)$, has inflection point(s) at

a) $x = -1$ only.

b) $x = -2$ and $x = 1$.

c) $x = -2$ only.

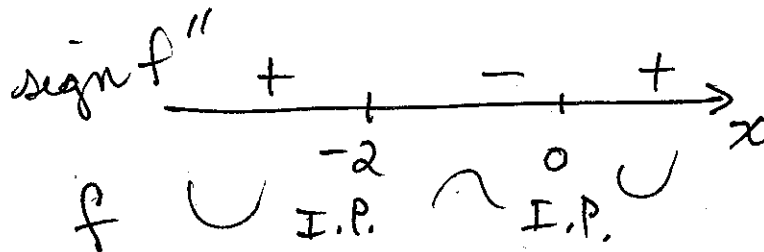
d) $x = -2$ and $x = 0$.

e) $x = 1$ only

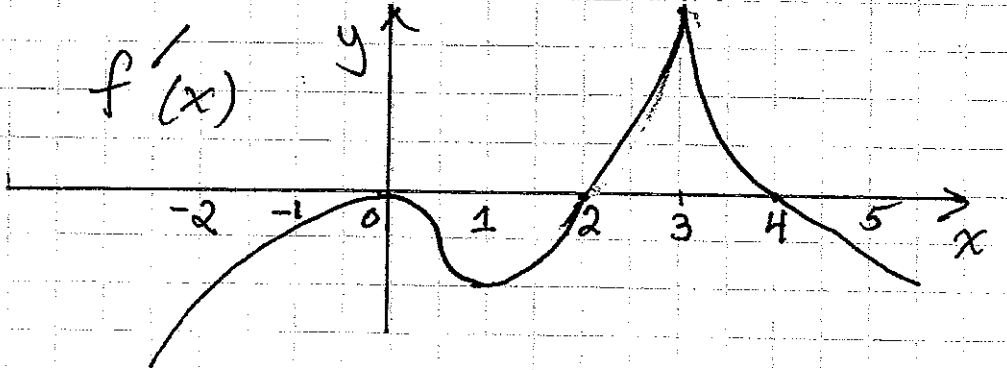
$$f''(x) = (x+2)^2 + 2(x-1)(x+2)$$

$$= (x+2) \left[x+2 + 2(x-1) \right]$$

$$= (x+2)(3x)$$

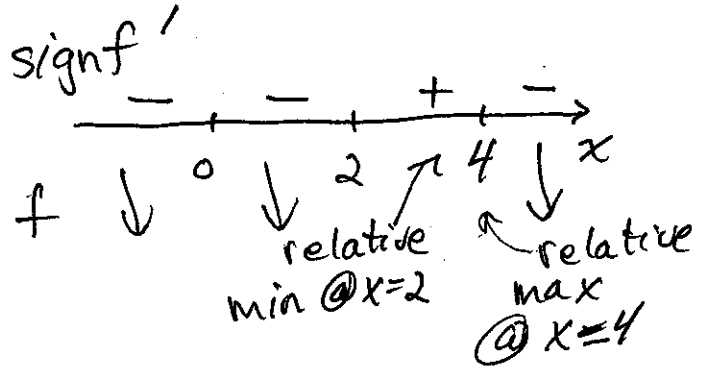


For problems 11 and 12, the graph of the derivative of $f(x)$ is shown.



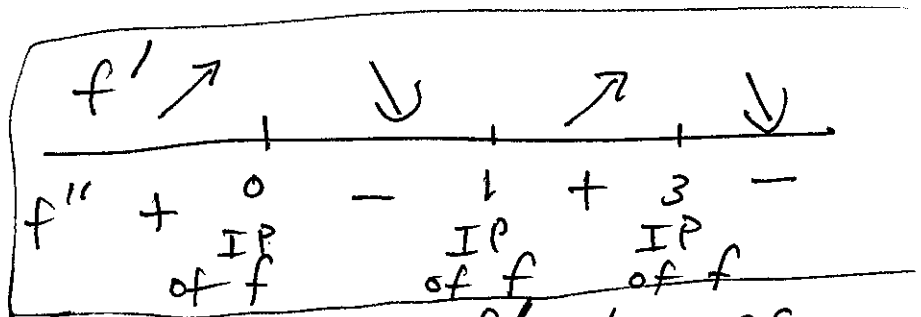
11. The function with the derivative shown has relative extrema

- a) min at $x = 2$ and max at $x = 4$.
- b) min at $x = 1$ and max at $x = 0$ and $x = 3$.
- c) min at $x = 4$ and max at $x = 2$.
- d) min at $x = 1$ only.
- e) none of these.



12. $f(x)$ has inflection point(s) at

- a) about $x = 0.5$.
- b) $x = 0$ and $x = 1$.
- c) $x = 0$, $x = 1$, and $x = 3$
- d) $x = 0$, $x = 2$, and $x = 4$.
- e) none of these.



Look where f' changes direction.

13. The derivative of $f(x) = \ln[(x^2 + e^x)(x-2)^3]$ is

$f'(x) =$

a) $\frac{2}{x} + \frac{3}{x-2}$

$f(x) = \ln(x^2 + e^x) + 3 \ln(x-2)$

$f'(x) = \frac{2x + e^x}{x^2 + e^x} + \frac{3}{x-2}$

b) $\frac{2x + e^x}{x^2 + e^x} + \frac{3}{x-2}$

c) $\frac{2x + e^x}{x^2 + e^x} + \frac{6}{x-2}$

d) $\frac{2}{x} + 1 + \frac{3}{x-2}$

e) none of these

14. The table shows the values of f' and f'' at three x -values.

x	0	1	2
$f'(x)$	0	1	0
$f''(x)$	-2	1	3

According to the 2nd derivative test, $f(x)$ has

The test only applies when $f'(a) = 0$ so only at $x=0$ and $x=2$.

a) a relative max at $x=0$ and relative min at $x=1$ and $x=2$.

At $x=0, f'' < 0$

b) a relative min at $x=0$ and relative max at $x=1$ and at $x=2$.

c) a relative max at $x=0$ and relative min at $x=2$.

relative max of f .

d) a relative max at $x=1$.

At $x=2, f'' > 0$

e) a relative min at $x=1$ and the test fails at $x=0$ and $x=2$.

relative min of f .

3. Find the derivative of each function. Do not simplify the derivative.

$$a) f(x) = \log \left[\frac{e^{x^2} + 1}{(x^2 + 2)^5} \right] = \log(e^{x^2} + 1) - 5 \log(x^2 + 2)$$

$$f'(x) = \left(\frac{2x}{e^{x^2} + 1} - 5 \cdot \frac{2x}{x^2 + 2} \right) \frac{1}{\ln 10}$$

$$= \left(\frac{2x}{e^{x^2} + 1} - \frac{10x}{x^2 + 2} \right) \frac{1}{\ln 10}$$

$$b) g(x) = \frac{e^{5x} + 1}{(x^2 + 4)^2}$$

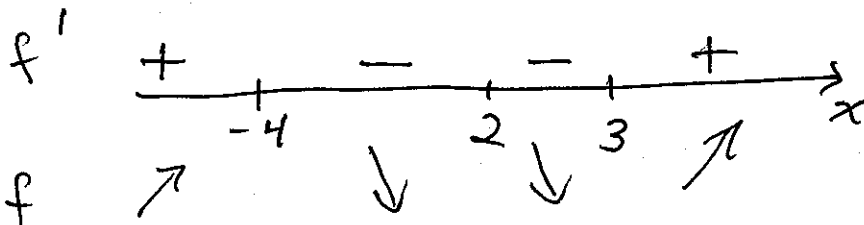
$$g'(x) = \frac{5e^{5x}(x^2 + 4)^2 - (e^{5x} + 1) \cdot 2(x^2 + 4)(2x)}{(x^2 + 4)^4}$$

4 pts

2. a) The derivative of a function, $f(x)$, is given by

$$f'(x) = (x-2)^2(x-3)(x+4)$$

Make a sign chart for $f'(x)$.



Answer with all possible x -values or state none exists.

$f(x)$ has a min at $x=3$ and a max at $x=-4$

$f(x)$ is increasing on $(-\infty, -4)$ and $(3, \infty)$

8 pts

b) The derivative of $g(x)$ is given by $g'(x) = x^2 e^{-x^3/12} = x^2 e^{-\frac{1}{12}x^3}$

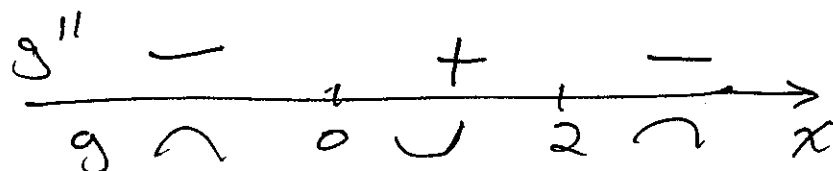
$g(x)$ has inflection point(s) at $x=0, x=2$

$g(x)$ is concave up on $(0, 2)$

$$\begin{aligned} g''(x) &= 2x e^{-\frac{1}{12}x^3} + x^2 \left(-\frac{3}{12}x^2\right) e^{-\frac{1}{12}x^3} \\ &= \left(2x - \frac{1}{4}x^4\right) e^{-\frac{1}{12}x^3} \\ &= x \left(2 - \frac{1}{4}x^3\right) e^{-\frac{1}{12}x^3} \end{aligned}$$

$$= 0 \text{ if } x=0 \text{ or } x=2$$

and changes sign at each

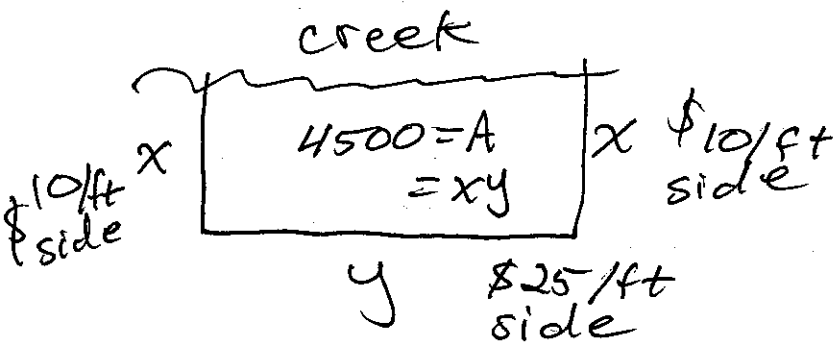


Test $x=1$ $g''(1) \geq 0$

Show work on all work out problems.

There are three work out problems. Each problem is 12 points.

1. A rectangular area of 4,500 square feet is to be bordered by a creek on one side and fencing on the other 3 sides. The material for the side opposite the creek costs \$25 per foot. The material for the other two sides costs \$10 per foot. Find the dimensions that will minimize the cost. After you find your answer, use the 2nd derivative test to show this gives the minimum cost.



$$\text{Cost} = 20x + 25y \quad \begin{array}{l} xy = 4500 \\ y = \frac{4500}{x} \end{array}$$

$$C(x) = 20x + \frac{25(4500)}{x}$$

$$C'(x) = 20 - \frac{25(4500)}{x^2}$$

$$\text{solve } C'(x) = 0. \quad 20 = \frac{25(4500)}{x^2}$$

$$20x^2 = 25(4500)$$

$$x = \sqrt{\frac{25(4500)}{20}} = 60 \quad y = \frac{4500}{60} = 75$$

$$C''(x) = \frac{25(4500)}{x^3} > 0 \text{ if } x = 60$$

at 60, min

15. For a certain product the demand quantity at price p dollars is $f(p) = e^{-.045p^2}$. Demand is inelastic for

- a) $0 < p < 3.33$ b) $0 < p < 11.11$ c) $p > 3.33$ d) $p > 11.11$ e) $0 < p < 5.55$

$$E(p) = \frac{-p(-.09pe^{-.045p^2})}{e^{-.045p^2}} = .09p^2$$

$$E(p) = 1 \text{ if } .09p^2 = 1 \quad p^2 = \frac{1}{.09} = \frac{100}{9}$$

$$p = \frac{10}{3} \approx 3.33$$

16. The elasticity function for a product is $E(p) = \frac{p^2}{256 + p^2}$.

If the price increases from \$12 to \$14 then the approximate per-cent change in demand is

a) 3.84% and revenue will decrease.

b) 3.76% and revenue will decrease.

c) 3.76% and revenue will increase.

d) 6% and revenue will decrease.

e) 6% and revenue will increase.

$E(p) < 1$ for all p
so revenue is always increasing.

$$E(12) \left(\frac{14-12}{12} \right) \times 100\%$$

$$= \frac{144}{400} \left(\frac{2}{12} \right) \times 100\%$$

$$= 6\%$$