

Math 142 Exam 2 Review

This is a sampling of past exam questions from several exams and is not a 'practice exam'.

You should also be able to look at a graph of a derivative and determine information about the original function such as intervals where the original is increasing/decreasing, concavity, local extreme points and inflections points. This can be practiced as follows: Start with a function $f(x)$. Find its derivative and enter it into your calculator. Graph it. Write all info about $f(x)$. Then graph $f(x)$ and observe.

1. Find the derivative with respect to x for $f(x)=$:

a) $(4x + 1)^3 e^{5x}$

b) $7^{\sqrt{3x^2+8}}$

c) $\frac{\log_4 \sqrt{x}}{x^6 + 1}$

d) $\ln \left[\frac{(4x^3 - 6)^2 (2x + 1)^3}{\sqrt{5x + 3}} \right]$ Use log rules or a problem like this will not be graded.

2. A profit function is $P(x) = -0.3x^2 + 182x - 1560$ where x is the quantity produced and sold.

a) Find the marginal profit function.

b) Use the marginal profit function to approximate the change in profit if production increases from 370 to 371.

3. A cost function is given by $C(x) = 0.5x^2 + 190x + 4548$. Find the production level that minimizes the average cost. Then find the values of the marginal cost and the average cost at this x . They should be equal.

4. P dollars is invested at annual interest rate 5.6% compounded continuously for t years.

a) Write the formula for the accumulated amount after t years.

b) Find the rate of change with respect to t of this accumulated amount.

c) Find the **relative** rate of change, with respect to t , of the accumulated amount.

Relative rate of change is $\frac{A'(t)}{A(t)}$.

5. For a certain product, the demand quantity, x , and price/unit, p , are related by $x^2 + 0.25 p^2 = 3600$.

a) Find the elasticity function, $E(p)$. On what price range is demand inelastic?

b) Revenue is $R(p) = pf(p)$. Show that revenue is a max when $E(p)=1$ for this case.

6. An elasticity function is $E(p) = 0.01 p^2$.

a) At what price is revenue at a maximum?

b) On what price range is demand inelastic? Is revenue increasing or decreasing on this price range?

c) Approximate the % decrease in demand if price increases from \$8 to \$9.

7. The derivative of $f(x)$ is $f'(x) = x^3(x+4)^2$.

Show all work including sign charts.

a) Determine the intervals where f is increasing and where f is decreasing. Locate any local max or min of f .

b) Determine the intervals where f is concave up/concave down. Locate any inflection points of f .

8. The derivative of $f(x)$ is $f'(x) = \frac{x^2}{x-3}$.

a) Find $f''(x)$.

b) Make a sign chart and locate any inflection point(s) of f .

9. For each value of a , tell what the 2nd derivative test says, if anything, about $f(x)$ at $x=a$. "No conclusion" and "does not apply" might be answers.

a	1	2	3	4
$f'(a)$	0	0	1	0
$f''(a)$	1	-2	4	0

10. Find the absolute max and the absolute min of the given function on the given interval:

a) $f(x) = x^3 - 3x + 10$ on the interval $[0, 2]$

- b) $f(x) = x^3 - 9x^2 + 15x$ on the interval $[-1, 2]$
 c) $f(x) = x^3 - 3bx^2$ on the interval $[0, 4b]$
 d) $f(x) = x^3 - 3a^2x$ on the interval $[-2a, 3a]$

11. The demand quantity, x , for a certain product is 0 if the price per unit is \$40 or more. The quantity demanded changes at the constant rate of 100 more units per \$3 decrease in the price. The fixed cost is \$2000 and the total cost increases at the constant rate of \$12 per unit.

- a) Find the profit function, $P(x)$.
 b) Find the approximate change in profit if x increases from 200 to 201.
 c) Find the marginal average profit function.

12.

Find the derivative of each.

a) $f(x) = \frac{x^4}{e^{5x} + 1}$

b) $f(x) = 8^{\sqrt{x}}$

c) $f(x) = e^{x^2 - 3x + 5}$

d) $f(x) = \ln \left[\frac{(e^x + 7)^4 (x^2 + 6x)}{(x + 2)^{1/3}} \right]$

In d, you must use log rules to receive credit. No reminder will be given on the exam.

13.

$$f(u) = u^7$$

$$\frac{du}{dx} = \frac{5x}{x^3 + 1} \text{ and } u(3) = 1$$

Find $\frac{df}{dx}$ at $x=3$.

14. The elasticity function for a certain product is given by $E(p) = \frac{3p^2}{2500 - p^2}$ where p is the price per unit.

a) At what price is revenue at a maximum?

b) On what price range is demand **inelastic**?

c) If the price is increased from \$20 to \$21, will revenue increase or decrease?

d) What is the approximate per-cent change in demand quantity if price increases from \$20 to \$21? Does demand increase or decrease with this price change?

15. The demand quantity for a certain product is given by $f(p) = 2000pe^{-p^2}$.

a) Find the relative rate of change of f .

b) Find the elasticity function, $E(p)$.

16. The derivative of $f(x)$ is given by $f'(x) = (x+3)^2(x-4)^3$.

a) At what x -value(s) is the tangent line to $f(x)$ horizontal?

b) List and classify all extreme points of $f(x)$. (Just the x -value(s)). You must show the sign chart.

c) List all inflection points of $f(x)$. *You must show the 2nd derivative and the sign chart.*

17. a) The derivative of $f(x)$ is given by $f'(x) = \frac{e^{-x}}{(x+2)^2}$. f is continuous except at $x = -2$.

a) Find $f''(x)$ and simplify.

b) List all inflection points of $f(x)$. *Note that the concavity changes at $x = -2$ but there is no point there so it is not called an inflection point.*

c) On what interval(s) is $f(x)$ concave up?

18.

Find the derivative of each.

a) $f(x) = 8^{x^2 - 5x + 12}$

b) $f(x) = \ln \left[\frac{(x^2 + 6x)(x - 4)^{\frac{1}{2}}}{(e^x + 1)^3} \right]$

19. The elasticity function for a certain product is given by $E(p) = \frac{p^2}{675 - 2p^2}$ where p is the price per unit.

a) At what price is revenue at a maximum?

b) On what price range is demand **inelastic**?

c) If the price is increased from \$12 to \$13, will revenue increase or decrease? what about demand?

d) What is the approximate per-cent change in demand quantity if price increases from \$12 to \$13? Round to 2 decimal places.

20. The derivative of $f(x)$ is given by $f'(x) = (x + 2)^2(x - 6)^3$.

a) At what x-value(s) is the tangent line to $f(x)$ horizontal?

b) List and classify all extreme points of $f(x)$. (Just the x-value(s)). You must show the sign chart.

c) List all inflection points of $f(x)$. *You must show the 2nd derivative and the sign chart.*

21. The derivative of $f(x)$ is given by $f'(x) = \frac{x^2}{(x^2 + 72)^2}$.

a) Find $f''(x)$ and simplify.

b) List all inflection points of $f(x)$. *You must show the appropriate sign chart.*

c) On what interval(s) is $f(x)$ concave up?

22. Two hundred pounds of fruit are being stored in a refrigeration unit. The quantity is decreasing by 10 pounds per week due to aging. The current price for the fruit is \$1.00 per pound. The price per pound is increasing by \$0.25 per week. After how many weeks will revenue be at a maximum? How do you know it is a max?

23. A box with square top and bottom has a volume of 1350 cubic feet. Material for the top and bottom costs \$2 per square foot and material for the sides costs \$5 per square foot. Find the dimensions of the box that minimize the cost of material. **Verify that your answer minimizes the cost by using the 2nd derivative test.**