

Math 142 Exam 3 Review

1. Find each antiderivative:

a)  $\int x^2(x^2 + 5x) dx$       b)  $\int \frac{x^2 + 7x^{3/2} + 10}{x} dx$

c)  $\int \frac{12x^2 + 8}{x^3 + 2x + 9} dx$       d)  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

e)  $\int \frac{e^x + 2}{(e^x + 2x + 1)^{1/3}} dx$       f)  $\int \frac{x + 3}{(x + 4)^2} dx$

2. A marginal average cost function is  $MAC = 0.3 - \frac{500}{x^2}$ . The average cost per unit if 50 units are produced is \$72. Find the total cost function.

3. Find  $y(t)$  if a)  $\frac{dy}{dt} = (t^2 + 2)e^{t^3 + 6t}$      $y(0) = 4$

b)  $\frac{dy}{dt} = \frac{1}{1-t}$     for  $t \neq 1$      $y(0) = 5$

4. Find the left and right hand Riemann sums with  $n$  equal subintervals.

a)  $f(x) = 16^x$     on  $[1, 2]$      $n = 4$

b)  $f(x) = 4 - x^2$     on  $[1, 3]$      $n = 4$

c)  $f(x) = 5x^3$     on  $[0, 2]$      $n = 5$

5. Show that  $F(x) = x \ln x - x$  is an antiderivative of  $f(x) = \ln x$  and evaluate

$$\int_1^c \ln x \, dx .$$

6. a) Evaluate  $\int_{-7}^2 f(x) dx$      $f(x) = \begin{cases} -3 & x < -4 \\ 1.5x + 3 & -4 \leq x \leq -2 \\ \sqrt{4 - x^2} & -2 \leq x \leq 2 \end{cases}$

b) Find the total area between  $f(x)$  (from part a) and the  $x$ - axis for  $x$  between  $-7$  and  $2$ .

7. Evaluate each definite integral.

$$a) \int_{-1}^3 (Kx^2 + Lx^3) dx \quad b) \int_0^T x\sqrt{x^2 + 4} dx \quad c) \int_0^{\ln A} e^t \sqrt{e^t + 3} dt$$

8.  $f(x) = \frac{x}{x^2 + 1}$  Find the average value of  $f(x)$  on a)  $[0, 4]$

b)  $[-4, 4]$

9. A cost function is  $C(x) = 15x + 2500$ .

a) Find the average cost per unit if 50 units are produced.

b) Find the average value of the total cost on  $[0, 50]$ .

10.  $f(x) = 2x^3 - 7x^2 - 9x$     $g(x) = x^3 - 3x^2 - 4x$

a) Find the area between  $f(x)$  and  $g(x)$ .

b) Find the area between  $f(x)$  and  $g(x)$  for  $x$  between 1 and 6.

11. Find the consumer surplus and the producer surplus at equilibrium.

a)  $D(x) = 25 - 0.1x$     $S(x) = 10 + 0.5\sqrt{x}$

b)  $D(x) = \sqrt{400 - 0.2x}$     $S(x) = \sqrt{50 + 0.3x}$

c)  $D(x) = 50 - 0.1x$     $S(x) = 9 + 0.1x$