Math 142 Exam 3 Review Solutions (out of class review)

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1. a) $\int x^4+5x^3\,dx = \boxed{\frac{1}{5}x^5+\frac{5}{4}x^4+C}$

b) $\int x+7x^{1/2}+\frac{10}{x}\,dx = \boxed{\frac{1}{2}x^2+\frac{14}{3}x^{3/2}+10\ln|x|+C}$

c) $u = x^3+2x+9$
$du = (3x^2+2)\,dx$
$4\,du = (12x^2+8)\,dx$

$\int \frac{4\cdot 1}{u}\,du = 4\ln|u|+C$
$= \boxed{4\ln|x^3+2x+9|+C}$

d) $u = x^{1/2}$
$du = \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}\,dx$
$2\,du = \frac{1}{\sqrt{x}}\,dx$

$\int 2e^u\,du$
$= 2e^u + C$
$= \boxed{2e^{\sqrt{x}}+C}$

e) $u = e^x+2x+1$
$du = (e^x+2)\,dx$

$\int \frac{1}{u^{1/3}}\,du = \int u^{-1/3}\,du$
$= \frac{3}{2}u^{2/3}+C$
$= \boxed{(e^x+2x+1)^{2/3}+C}$

1 f) $u = x+4$ $\quad \dfrac{x+3}{(x+4)^2}\,dx = \dfrac{x+3}{u^2}\,du$

$du = dx$

we have to solve for $x+3$.

$u = x+4 \longrightarrow x = u-4 \longrightarrow x+3 = u-1$

$$\int \frac{u-1}{u^2}\,du = \int \frac{1}{u} - u^{-2}\,du$$

$$= \ln|u| + u^{-1} + C = \boxed{\ln|x+4| + \frac{1}{x+4} + C}$$

2. $\left(\dfrac{C(x)}{x}\right)' = .3 - \dfrac{500}{x^2} = .3 - 500x^{-2}$

$\dfrac{C(x)}{x} = .3x + \dfrac{500}{x} + C$

$\dfrac{C(50)}{50} = 72$ (given) $= .3(50) + \dfrac{500}{50} + C$

$= 15 + 10 + C$

$72 - 25 = C$

$= 47 = C$

$\dfrac{C(x)}{x} = .3x + \dfrac{500}{x} + 47$

$$\boxed{C(x) = .3x^2 + 500 + 47x}$$

3. a) $u = t^3 + 6t$

$du = (3t^2 + 6)\,dt = 3(t^2 + 2)\,dt$

$\frac{1}{3}du = (t^2 + 2)\,dt$

$\frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + c = \frac{1}{3}e^{t^3+6t} + c$

$y(0) = 4 = \frac{1}{3}e^0 + c = \frac{1}{3} + c$
given

$c = 4 - \frac{1}{3} = \frac{11}{3}$

$$\boxed{y(t) = \frac{1}{3}e^{t^3+6t} + \frac{11}{3}}$$

b) $u = 1 - t$ $\quad \int -\frac{1}{u}\,du = -\ln|u| + c$

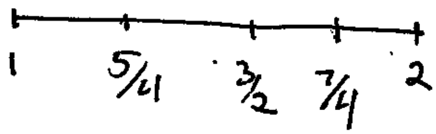
$du = -dt$

$-du = dt$ $\quad = -\ln|1-t| + c$

$y(0) = 5 = -\ln 1 + c = c$
given

$y(t) = -\ln|1-t| + 5$

4. a) $\Delta x = \frac{1}{4}$



$$L_4 = \frac{1}{4}\left[16^1 + 16^{5/4} + 16^{3/2} + 16^{7/4}\right]$$

$$= \frac{1}{4}\left[16 + 32 + 64 + 128\right]$$

$$= \boxed{60}$$

$$R_4 = \frac{1}{4}\left[32 + 64 + 128 + 256\right]$$

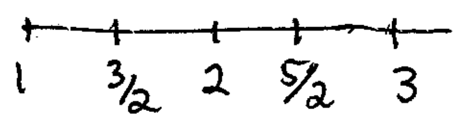
$$= \boxed{120}$$

Note $\frac{L_4 + R_4}{2} = 90$

$$\int_1^2 16^x \, dx = \left.\frac{16^x}{\ln 16}\right|_1^2 = \frac{240}{\ln 16}$$

$$\approx 86.56$$

b) $\Delta x = \frac{3-1}{4} = \frac{1}{2}$



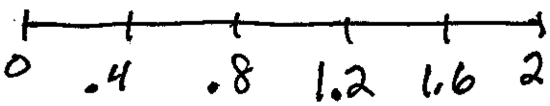
$$L_4 = \frac{1}{2}\left[(4-1) + (4 - 9/4) + (4-4) + \left(4 - \frac{25}{4}\right)\right]$$

$$= \frac{1}{2}\left[2.5\right] = 1.25$$

$$R_4 = \frac{1}{2}\left[4 - 9/4 + 4 - 4 + 4 - \frac{25}{4} + (-5)\right] = -2.75$$

$(L_4 + R_4)/2 = -.75 \quad \int_1^3 4 - x^2 \, dx = -2/3$

4c) $\Delta x = \frac{2-0}{5} = .4$



$$L_5 = .4[5(0)^3 + 5(.4)^3 + 5(.8)^3 + 5(1.2)^3 + 5(1.6)^3]$$

$$= 12.8$$

$$R_5 = .4[5(.4)^3 + 5(.8)^3 + 5(1.2)^3 + 5(1.6)^3 + 5(2^3)]$$

$$= .4[32 + 40] = 28.8$$

$$\frac{L_5 + R_5}{2} = \frac{41.6}{2} = 20.8 \qquad \int_0^2 5x^3\,dx = \frac{5}{4}x^4\Big|_0^2$$

$$= 20$$

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use product rule on $x\ln x$

5. $F'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x} - 1 = \ln x$

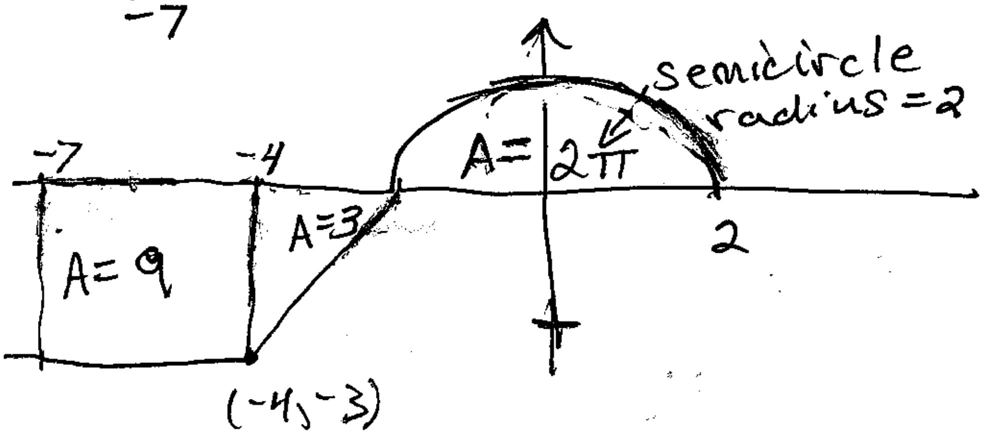
$$\int_1^c \ln x\,dx = F(c) - F(1)$$

$$= c\ln c - c - (0 - 1)$$

$$= c\ln c - c + 1$$

6a) $f(x) = \begin{cases} -3 & x < -4 \\ 1.5x+3 & -4 \le x < -2 \\ \sqrt{4-x^2} & -2 \le x \le 2 \end{cases}$

$\int_{-7}^{2} f(x)\,dx$ = area above x-axis − area below x-axis



$A_{above} = 2\pi \qquad A_{below} = 12$

$\int_{-7}^{2} f(x)\,dx = 2\pi - 12$

b) $\int_{-7}^{2} |f(x)|\,dx = 2\pi + 12$

A above + A below

7. a) $\int_{-1}^{3} Kx^2 + Lx^3 \, dx$

$= \frac{1}{3}Kx^3 + \frac{1}{4}Lx^4 \Big|_{-1}^{3}$

$= 9K + \frac{81}{4}L - \left(-\frac{1}{3}K + \frac{1}{4}L\right)$

$= \frac{35}{4}K + 20L$

b) $u = x^2 + 4$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int \frac{1}{2} u^{1/2} du = \frac{1}{3} u^{3/2}$

$= \frac{1}{3}(x^2+4)^{3/2} \Big|_0^T$

$= \frac{1}{3}(T^2+4)^{3/2} - \frac{1}{3}(4^{3/2})$

$= \boxed{\frac{1}{3}(T^2+4)^{3/2} - \frac{8}{3}}$

c) $u = e^t + 3$

$du = e^t\,dt$

$\int u^{1/2} du = \frac{2}{3} u^{3/2}$

$= \frac{2}{3}(e^t+3)^{3/2} \Big|_0^{\ln A}$

$= 2/3(A+3)^{3/2} - \frac{2}{3}(3^{3/2})$

8\. a) $\frac{1}{4}\int_0^4 \frac{x}{x^2+1}\,dx$

$= \frac{1}{4}\left(\frac{1}{2}\right)\ln(x^2+1)\Big|_0^4$

$= \boxed{\frac{1}{8}\ln 17}$

$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln u + C$

$= \frac{1}{2}\ln(x^2+1)$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

b) $\frac{1}{8}\int_{-4}^{4} \frac{x}{x^2+1}\,dx = \frac{1}{8}\cdot\frac{1}{2}\ln(x^2+1)\Big|_{-4}^{4} = 0$

9\. a) $\frac{C(50)}{50} = \frac{15(50)+200}{50} = \$19$ per unit

b) $\frac{1}{50}\int_0^{50}(15x+2500)\,dx = \frac{1}{50}\left[\frac{15}{2}x^2+2500x\Big|_0^{50}\right]$

$= 2875$

10\. a) By hand: $f(x)-g(x) = x^3-4x^2-5x$

$= x(x^2-4x-5)$

$= x(x-5)(x+1)$

$= 0$ at $x=0,\ x=5,\ x=-1$

$\left|\int_{-1}^{0}(x^3-4x^2-5x)\,dx\right| + \left|\int_0^5(x^3-4x^2-5x)\,dx\right|$

$= \left|\frac{1}{4}x^4-\frac{4}{3}x^3-\frac{5}{2}x^2\Big|_{-1}^{0}\right| + \left|\frac{1}{4}x^4-\frac{4}{3}x^3-\frac{5}{2}x^2\Big|_0^5\right| = 73.8\overline{3}$

10 b. $\left|\int_1^5 x^3-4x^2-5x\,dx\right| + \left|\int_5^6 x^3-4x^2-5x\,dx\right|$

$= |-69.\overline{3}| + |18.91\overline{6}| = 88.25$

Math 9 Math NUM enter (Y1 − Y2), x, 1, 6)

		Math 9(Y1 − $\bar{P}$, x, 0, $\bar{x}$)	Math 9($\bar{P}$ − Y2, x, 0, $\bar{x}$)
11.	Eq = $(\bar{x}, \bar{P})$	C.S.	P.S.
a)	(127.42359, 12.26) estimate	\$811.54	\$96.19
b)	(700, 16.124515) I used (700, 16.12)	\$1408.09	\$2753.29
c)	$(68.\overline{3},\ 29.50)$	\$700.42	\$700.42