

Math 142 Fall 2011 Exam 3 **Print Name** _____
class time or section _____

There are 16 multiple-choice problems worth 4 points each followed by 3 work out problems worth 12 points each. Partial credit will be given only if work is shown.

"An Aggie does not lie, cheat, or steal or tolerate those who do"

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Your signature _____

White Test

Math 142 Exam 3 F2011

Print Name Key

Section or class time _____

For problems 1 through 5, find the most general antiderivative.

$$1. \int \frac{3x^3 + 2x + 1}{x} dx = \int 3x^2 + 2 + \frac{1}{x} dx \\ = x^3 + 2x + \ln|x| + C$$

a) $x^3 + 2x + \ln|x| + C$ b) $\frac{3}{4}x^4 + x^2 + x - \ln|x| + C$

c) $\frac{3}{2}x^2 + 2 + \frac{2}{x} + C$ d) $\frac{3}{4}x^4 + x^2 - \ln|x| + C$

e) $6x - \frac{1}{x^2} + C$

$$2. \int \frac{1}{x^2} + \frac{5}{\sqrt{x}} dx = \int x^{-2} + 5x^{-1/2} dx = -x^{-1} + 10x^{1/2} + C \\ = -\frac{1}{x} + 10\sqrt{x} + C$$

a) $6x + \frac{1}{x} + \frac{2}{3}x^{3/2} + C$ b) $-\frac{1}{x} + 10\sqrt{x} + C$

c) $-\frac{1}{3}x^3 + 6x - 2\sqrt{x} + C$ d) $\frac{3}{x^3} + \frac{15}{2x^{3/2}} + C$

e) none of these

$$3. \int 6(x+1)(x+2) = \int 6(x^2 + 3x + 2) dx$$

$$= \int 6x^2 + 18x + 12 dx = 2x^3 + 9x^2 + 12x + C$$

a) $\frac{3}{2}(x+1)^2(x+2)^2 + C$ b) $2x^3 + 3x^2 + 2x + C$

c) $x^2 + 3x + 2 + C$ **(d)** $2x^3 + 9x^2 + 12x + C$ e) $12x + 18 + C$

$$4. \int \frac{x^2 + 4x}{x^3 + 6x^2 + 10} dx$$

$u = x^3 + 6x^2 + 10$

$$du = (3x^2 + 12x) dx = 3(x^2 + 4x) dx$$

$$\frac{1}{3} du = (x^2 + 4x) dx \quad \int \frac{1}{3} \frac{du}{u} = \frac{1}{3} \ln|u| + C$$

a) $3 \ln|x^3 + 6x^2 + 10| + C$ b) $\frac{\frac{1}{3}x^3 + 2x^2}{\frac{1}{4}x^4 + 2x^3 + 10x} + C$

c) $\frac{-1}{2}(x^3 + 6x^2 + 10)^{-2} + C$ **(d)** $\frac{1}{3} \ln|x^3 + 6x^2 + 10| + C$

e) none of these

5. $\int e^x \sqrt[3]{2e^x + 1} dx$

a) $\frac{3}{8}(2e^x + 1)^{4/3} + C$ b) $\frac{3}{8}e^x(2e^x + 1)^{4/3} + C$

c) $\frac{3}{2}(2e^x + 1)^{4/3} + C$ d) $\frac{3}{2}e^x(2e^x + 1)^{4/3} + C$

e) none of these

$$u = 2e^x + 1 \quad \frac{1}{2} \int u^{4/3} du = \frac{3}{8} u^{7/3} + C$$

$$du = 2e^x dx \quad = \frac{3}{8} (2e^x + 1)^{7/3} + C$$

$$\frac{1}{2} du = e^x dx$$

6. A cost function is given by $C(x) = 0.3x^2 + 6x + 2000$ dollars for x units. Find the average value of the total cost for the first 50 units.

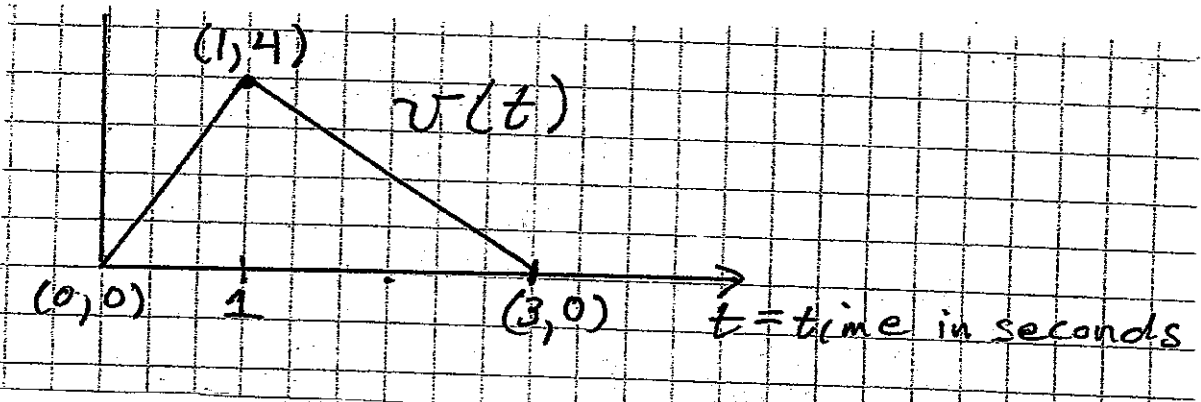
- a) \$112 b) \$3050 c) \$120,000 d) \$61 e) \$2400

$\frac{1}{50} \int_0^{50} .3x^2 + 6x + 2000 dx$ use calculator or

$\frac{1}{50} \left[.1x^3 + 3x^2 + 2000x \Big|_0^{50} \right]$

$= 2400$

7. An object traveling in a straight line has velocity at time t sec. as shown. The position at $t=0$ sec is 0, $s(0)=0$.



$$s'(t) = v(t) \text{ so } s(3) - s(0) = \int_0^3 v(t) dt$$

$$= \text{area above} - \text{area below} = \frac{1}{2}(3 \times 4) = 6$$

but all is above area of Δ

The position at time $t = 3$ sec is

$$s(3) - 0 = 6$$

since $s(0) = 0$ was given.

a) 0

b) 6

c) 12

d) 3

e) 4

8. Find L_3 , the left hand Riemann sum for the function $f(x) = 3(8^x)$ on the interval $[0, 1]$ using 3 equal subintervals.

a) 7

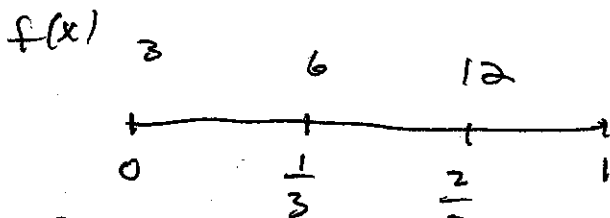
b) 14

c) 15

d) 21

e) 42

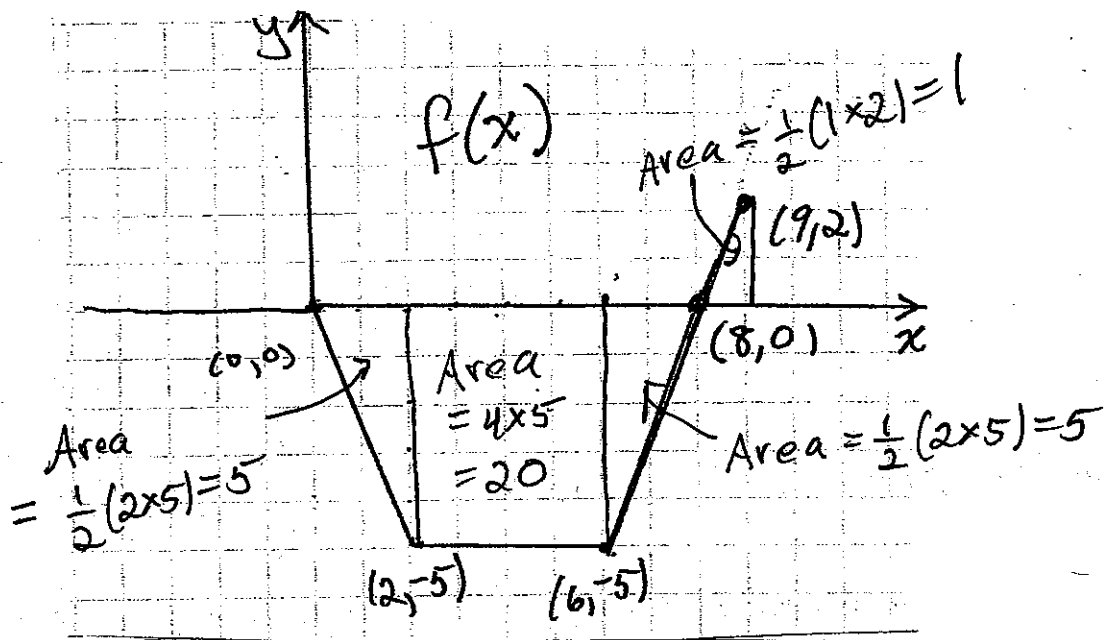
$$\Delta x = \frac{1-0}{3} = \frac{1}{3}$$



$$\frac{1}{3} [3 + 6 + 12] = 7$$

x	$3(8^x)$ left endpoints.
0	$8^0 = 1$ $3(8^0) = 3$
$\frac{1}{3}$	$8^{1/3} = 2$ $3(8^{1/3}) = 6$
$\frac{2}{3}$	$8^{2/3} = 4$ $3(8^{2/3}) = 12$

For problems 9 and 10, use the function, $f(x)$, shown in the graph.



$$\text{Area above} - \text{Area below} = 1 - 30 = -29$$

$$= \int_0^9 f(x) dx$$

9. $\int_0^9 f(x) dx$ is equal to

- a) 31 b) -31 c) 29 **d) -29** e) 1

$$\text{Area above} + \text{area below} = 1 + 30 = 31$$

$$= \int_0^9 |f(x)| dx$$

10. $\int_0^9 |f(x)| dx$ is equal to

- a) 30 **b) 31** c) 29 d) -30 e) 1

11. The position of an object at time $t = 0$ is $s(0) = -3$ ft. (It starts 3 feet left of home). Its velocity at $t = 0$ is $v(0) = 12$ ft/sec.

The acceleration is $a(t) = 9t + 2$ ft/sec².

The position at time $t = 2$ sec is

- a) 40 feet **b) 37 feet** c) 16 feet d) 43 feet e) none of these

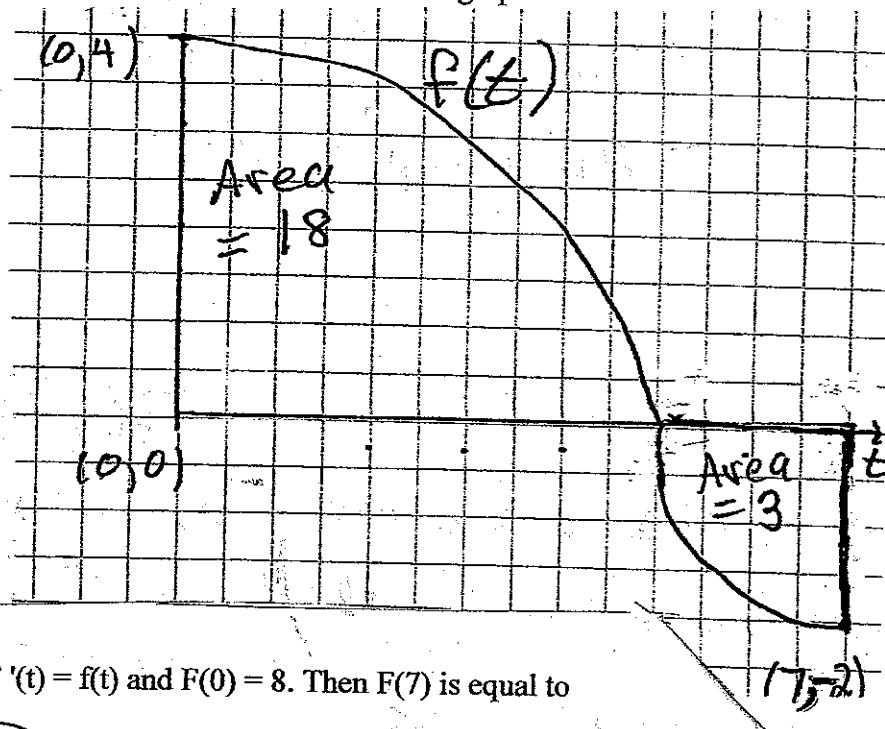
$$v(t) = \int a(t) dt = \frac{9}{2}t^2 + 2t + C \quad v(0) = 0 + 0 + C = 12$$

$$s(t) = \int v(t) dt = \frac{3}{2}t^3 + 2t + 12t + C_1 \quad 0 + 0 + 0 + C_1 = -3$$

$$s(2) = \frac{3}{2}(8) + 2(2) + 12(2) - 3$$

$$= 12 + 4 + 24 - 3 = 37$$

For 12 and 13, $f(t)$ is the function in the graph.



12. $F'(t) = f(t)$ and $F(0) = 8$. Then $F(7)$ is equal to

- a) 23** b) 30 c) 29 d) -2 e) 2

$$F(7) - F(0) = \int_0^7 f(t) dt = \text{area above} - \text{area below} = 18 - 3 = 15$$

Given $F(0) = 8$ so

$$F(7) - 8 = 15 \quad F(7) = 15 + 8 = 23$$

13. $F(7) - F(5)$ is equal to

- a) 3 **b) -3** c) -2 d) 15 e) -1

$$F(7) - F(5) = \int_5^7 f(t) dt = -3, \text{ minus since the area is below.}$$

14. $\int_1^B 6x(x+1)dx$ is equal to

a) $\frac{3}{2}B^2(B+1)^2 - 6$ b) $2B^3 + 3B^2$ c) $12B - 12$

d) $2B^3 + 3B^2 - 5$ e) $3(B+1)^2 - 12$

$$\int_1^B 6x^2 + 6x dx = 2x^3 + 3x^2 \Big|_1^B$$

$$= 2B^3 + 3B^2 - (2 + 3)$$

$$= 2B^3 + 3B^2 - 5$$

15. The area between the graphs of $f(x) = x^2 - 2x$ and $g(x) = 2x + 5$ is closest to which of the following numbers?

- a) 36 b) 36.24 c) 72 d) -36 e) 78.67

Math 9 (Math NUM enter
 fnint (abs(f(x)-g(x)), x, -1, 5))

or $\int_{-1}^5 |f(x) - g(x)| dx = 36$

$$f(x) - g(x) = x^2 - 4x - 5$$

$$= (x-5)(x+1)$$

= 0 at $x = -1$
 $x = 5$
 or use 2nd TRACE 5

16. Demand and Supply equations for a product are given by

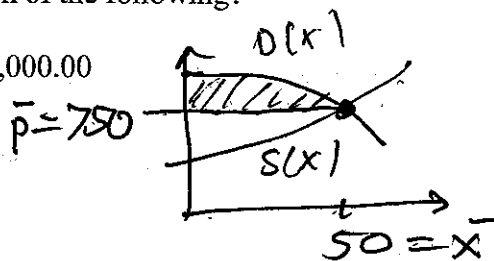
$D(x) = 1000 - 0.1x^2$ $S(x) = 250 + 0.2x^2$

Use Calculator

The consumers' surplus at equilibrium is closest to which of the following?

- a) \$16666.67 b) \$33,333.33 c) \$13,350,000.00
 d) \$25000 e) \$8333.33

$\int_0^{50} D(x) - 750 dx$



Work out section. Each problem is 12 points. Show all work on each problem.

1. $F'(x) = \frac{15(x-1)}{(x+1)^{1/3}}$ $F(0) = 8$ Find $F(x)$.

Show all work.

$$u = x+1$$
$$du = dx$$

$$15 \int \frac{x-1}{u^{1/3}} du$$

Solve for x
 $u = x+1$
 $\rightarrow u-1 = x$

$$u-1-1 = x-1$$

$$15 \int \frac{u-2}{u^{1/3}} du$$

Distribute $u^{-1/3}$

$$= 15 \int (u-2)u^{-1/3} du = 15 \int u^{2/3} - 2u^{-1/3} du$$

$$= 15 \left(\frac{2}{5} u^{5/3} - u^{2/3} \right) + C$$

$$= 6(x+1)^{5/3} - 45(x+1)^{2/3} + C$$

Solve for C :

$$6(0+1)^{5/3} - 45(0+1)^{2/3} + C = 8$$

$$6 - 45 + C = 8$$

$$C = 47$$

$$F(x) = 6(x+1)^{5/3} - 45(x+1)^{2/3} + 47$$

2. Evaluate by hand, showing all work. Use the fundamental theorem of calculus, not Riemann sums.

$$\int_1^5 3(x-1)\sqrt{x^2-2x+10} dx \quad \text{Find antiderivative?}$$

$$u = x^2 - 2x + 10$$

$$du = (2x - 2) dx$$

$$du = 2(x - 1) dx$$

$$\frac{1}{2} du = (x - 1) dx$$

$$\frac{3}{2} \int u^{1/2} du$$

$$= \frac{3}{2} \cdot \frac{2}{3} u^{3/2} + C$$

$$= u^{3/2} + C$$

$$(x^2 - 2x + 10)^{3/2} \Big|_1^5$$

$$= (25)^{3/2} - (9)^{3/2}$$

$$= 125 - 27$$

$$= 98$$

If you put limits on the u integral they should be

$$u(1) = 1^2 - 2 + 10 = 9$$

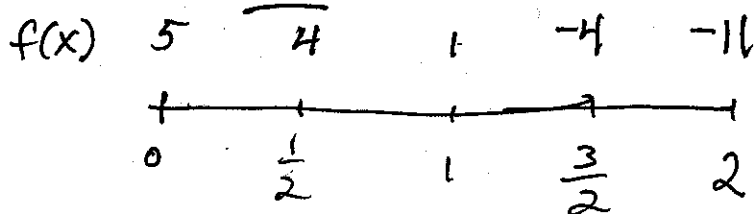
$$u(5) = 5^2 - 10 + 10 = 25$$

You did not lose pts. for this but it was circled in red.

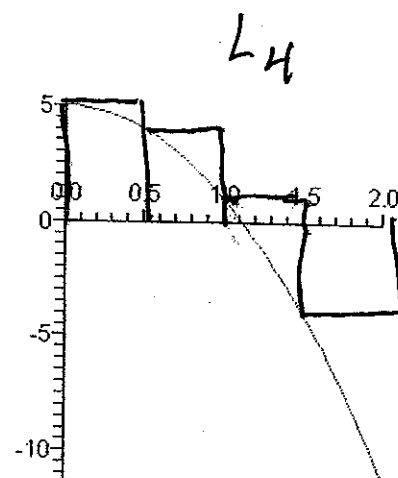
3. Find the average of the left and right hand Riemann sums for $f(x) = 5 - 4x^2$ on the interval $[0, 2]$ using 4 equal subintervals. Compare to $\int_0^2 f(x) dx$, rounded to 3 decimal places. Write the answers below.

$$\Delta x = \frac{2-0}{4} = \frac{1}{2}$$

Find and sketch L_4 showing all work.

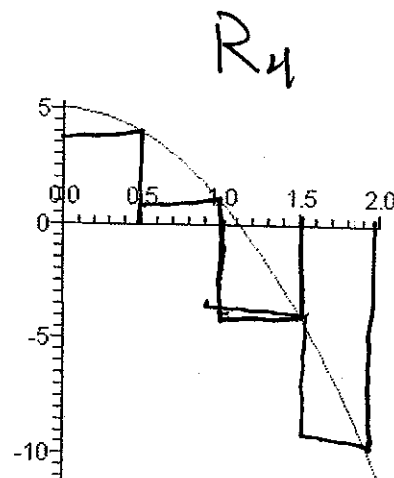


$$L_4 = \frac{1}{2} [5 + 4 + 1 - 4] = 1$$



Find and sketch R_4 showing all work.

$$R_4 = \frac{1}{2} [4 + 1 - 4 - 11] = -5$$



$$\frac{L_4 + R_4}{2} = \frac{-5 + 1}{2} = -2$$

edit \rightarrow $\int_0^2 f(x) dx = -\frac{2}{3}$ or $-0.666\dots$

typo \rightarrow You may use the calculator for the true value of the integral.

