Math 142 Fall 2011 Exam 3 **Print** Name____________________

class time or section____________________

There are 16 multiple-choice problems worth 4 points each followed by 3 work out problems worth 12 points each. Partial credit will be given only if work is shown.

"An Aggie does not lie, cheat, or steal or tolerate those who do"

"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work."

Your signature ______________________________

B

Yellow Test

Math 142 Exam 3 F2011 Print Name Key
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For problems 1 through 5, find the most general antiderivative.

1. $\int \frac{1}{x^2}+\frac{5}{\sqrt{x}}dx = \int x^{-2}+5x^{-1/2}\,dx = -x^{-1}+10x^{1/2}+C$

$= \frac{-1}{x}+10\sqrt{x}+C$

$a)\ 6x+\frac{1}{x}+\frac{2}{3}x^{3/2}+C$ $b)\ \frac{-1}{x}+10\sqrt{x}+C$

$c)\ -\frac{1}{3}x^3+6x-2\sqrt{x}+C$ $d)\ \frac{3}{x^3}+\frac{15}{2x^{3/2}}+C$

e) *none of these*

2. $\int \frac{3x^3+2x+1}{x}dx = \int 3x^2+2+\frac{1}{x}\,dx$

$= x^3+2x+\ln|x|+C$

$a)\ x^3+2x+\ln|x|+C$ $b)\ \frac{3}{4}x^4+x^2+x-\ln|x|+C$

$c)\ \frac{3}{2}x^2+2+\frac{2}{x}+C$ $d)\ \frac{3}{4}x^4+x^2-\ln|x|+C$

$e)\ 6x-\frac{1}{x^2}+C$

$$6x^2+18x+12$$

3. $\int 6(x+1)(x+2) = \int 6(x^2+3x+2)\,dx$

$$= 2x^3+9x^2+12x+C$$

a) $2x^3+3x^2+2x+C$   b) $\frac{3}{2}(x+1)^2(x+2)^2+C$

c) $x^2+3x+2+C$   d) $12x+18+C$

e) $2x^3+9x^2+12x+C$

4. $\int e^x \sqrt[3]{2e^x+1}\,dx$

$u=2e^x+1$

$du=2e^x\,dx$

$\frac{1}{2}du=e^x\,dx$

$\frac{1}{2}\int u^{1/3}\,du$

$= \frac{1}{2}\cdot\frac{3}{4}u^{4/3}+C$

$= \frac{3}{8}(2e^x+1)^{4/3}+C$

a) $\frac{3}{8}e^x(2e^x+1)^{4/3}+C$   b) $\frac{3}{8}(2e^x+1)^{4/3}+C$

c) $\frac{3}{2}(2e^x+1)^{4/3}+C$   d) $\frac{3}{2}e^x(2e^x+1)^{4/3}+C$

e) *none of these*

5. $\int \frac{x^2+4x}{x^3+6x^2+10}dx$

$u = x^3 + 6x^2 + 10$

$du = (3x^2 + 12x)\,dx$

$\frac{1}{3}du = (x^2+4x)\,dx$

$\frac{1}{3}\int \frac{du}{u}$

$= \frac{1}{3}\ln|u| + C$

$a)\ 3\ln|x^3+6x^2+10|+C$ $\quad$ $b)\ \frac{\frac{1}{3}x^3+2x^2}{\frac{1}{4}x^4+2x^3+10x}+C$

$c)\ \frac{-1}{2}(x^3+6x^2+10)^{-2}+C$ $\quad$ $d)\ \frac{1}{3}\ln|x^3+6x^2+10|+C$

*e) none of these*

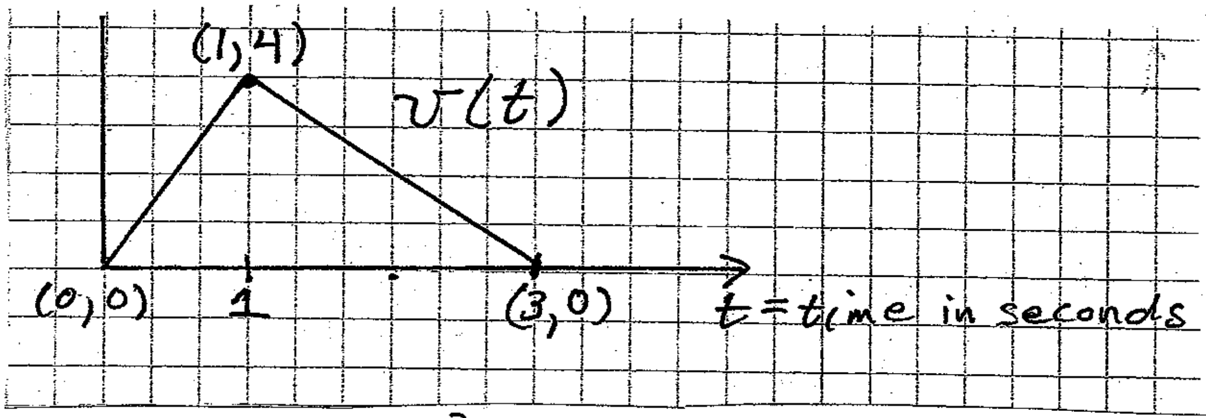
6. A cost function is given by $C(x) = 0.3x^2 + 12x + 2000$ dollars for $x$ units. Find the average value of the total cost for the first 50 units.

a) \$232 $\quad$ b) \$3170 $\quad$ c) \$127,500 $\quad$ d) \$2550 $\quad$ e) \$67

$\frac{1}{50}\int_0^{50} .3x^2 + 12x + 2000\ dx$ = use calculator or

$= \frac{1}{50}\left[ .1x^3 + 6x^2 + 2000x \Big|_0^{50} \right] = 2550$

7. An object traveling in a straight line has velocity at time t sec. as shown. The position at t=0 sec is 0, s(0)=0.




$$s(3) = s(3) - s(0) = \int_0^3 s'(t)\,dt = \int_0^3 v(t)\,dt$$

since $s(0) = 0$

$= \text{area above} - \text{area below} = \frac{1}{2}(3 \times 4) = 6$

but all is above

area of $\Delta$

The position at time t = 3 sec is

a) 4 b) 0 c) 3 d) 12 e) 6

8. Find $R_3$, the **right** hand Riemann sum for the function $f(x) = 3(8^x)$ on the interval [0, 1] using 3 equal subintervals.

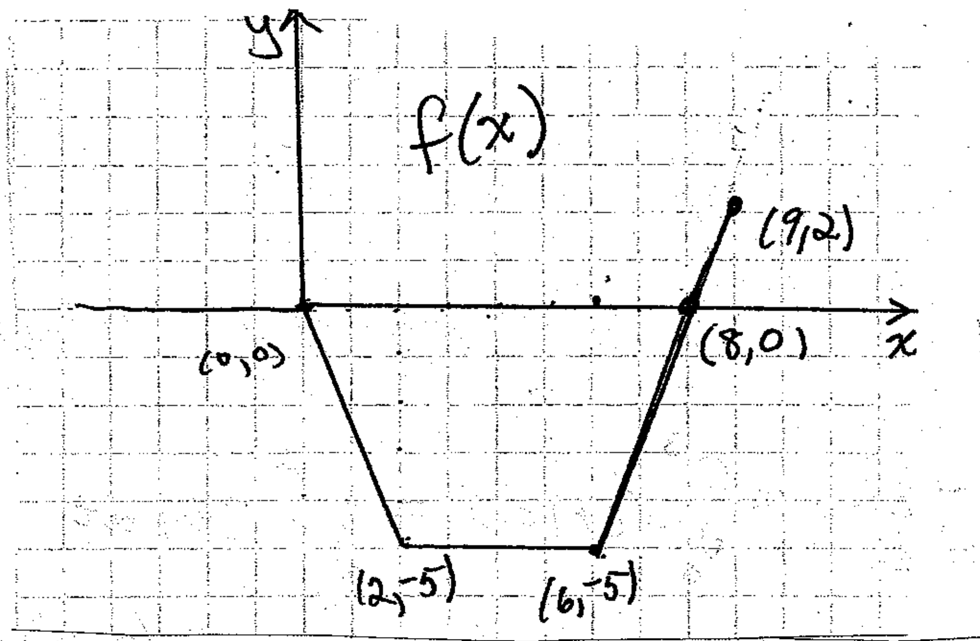
a) 7 b) 14 c) 15 d) 21 e) 42

| $x$ | $3(8^x)$ |
|---|---|
| 0 | 3 |
| 1/3 | 6 |
| 2/3 | 12 |
| 1 | 24 |

$R_3 = \frac{1}{3}[6 + 12 + 24]$

$= 14$

For problems 9 and 10, use the function, *f(x)*, shown in the graph.




9. $\int_0^9 f(x)dx$ is equal to

a) 1 b) 31 c) -31 d) 29 e) -29

10. $\int_0^9 | f(x) | dx$ is equal to

a) 1 b) 30 c) 31 d) 29 e) -30

11. The position of an object at time t = 0 is $s(0) = -3$ ft. (It starts 3 feet left of home). Its velocity at $t = 0$ is $v(0) = 12$ ft/sec.

The acceleration is $a(t) = 9t + 2$ ft/sec$^2$.
The position at time $t = 2$ sec is

a) 157 feet  b) 37 feet  c) 92 feet  d) 89 feet  e) none of these

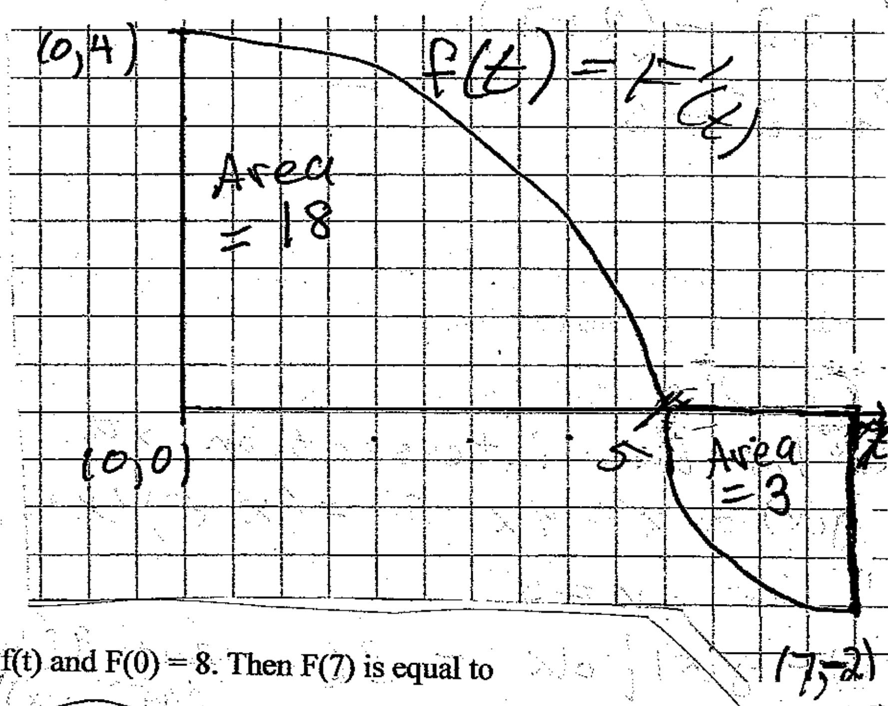
$v(t) = \int a(t)\,dt = \frac{9}{2}t^2 + 2t + c \quad v(0) = 12 \quad 0 + 0 + c = 12$

$s(t) = \int v(t)\,dt = \frac{3}{2}t^3 + t^2 + 12t + C_1 \quad s(0) = -3 = 0 + 0 + C_1$

$s(t) = \frac{3}{2}t^3 + t^2 + 12t - 3$

Plug in $t = 2$

For 12 and 13, f(t) is the function in the graph.




12. F '(t) = f(t) and F(0) = 8. Then F(7) is equal to

a) 2  b) 23  c) 30  d) 29  e) - 2

$F(7) - F(0) \underset{\uparrow}{=} \int_0^7 f(t)\,dt = 18 - 3 = 15$

since $F' = f$

since $F(0) = 8$

$F(7) - 8 = 15$

$F(7) = 23$

13. F(7) - F(5) is equal to

a) -1  b) -3  c) 3  d) -1  e) 15

$F(7) - F(5) = -3$  minus since the area is below the x-axis

14. $\int_1^B 6x(x+1)dx$ is equal to

$a)\ \frac{3}{2}B^2(B+1)^2-6$ $\quad b)\ 2B^3+3B^2-5$ $\quad c)\ 12B-12$

$d)\ 2B^3+3B^2$ $\quad e)\ 3(B+1)^2-12$

$$\int_1^B 6x^2+6x\,dx = 2x^3+3x^2\Big|_1^B$$

$$=2B^3+3B^2-(2+3)$$

$$=2B^3+3B^2-5$$

15. The area between the graphs of $f(x)=x^2-2x$ and $g(x)=2x+5$ is closest to which of the following numbers?

a) 78.67 b) 36 c) 36.24 d) 72 e) -36

Solve for the intersection pts.

$f(x)-g(x) = x^2-4x-5 = (x-5)(x+1) = 0$ if

$x=-1$ or $x=5$

$\int_{-1}^{5} |f(x)-g(x)|\,dx = 36$ Math 9 (Math NUM enter f(x) − g(x)), x, −1, 5)

16. Demand and Supply equations for a product are given by

$D(x)=1000-0.1x^2$ $\qquad S(x)=250+0.2x^2$ $\qquad (\bar{x},\bar{p})=(50,750)$

The consumers' surplus at equilibrium is closest to which of the following?

a) $16666.67 b) $8,333.33 c) $13,350,000.00

d) $25000 e) $33,333.33

C.S. = Math 9 ($D(x)-\bar{p}$ 750, $x$, 0, $\bar{x}$ 50) = 8333.33

Work out section. Each problem is 12 points. Show all work on each problem.

1. Evaluate by hand, showing all work. Use the fundamental theorem of calculus, not Riemann sums. $\int_1^5 3(x-1)\sqrt{x^2-2x+10}\,dx$

$u = x^2 - 2x + 10$

$du = (2x-2)\,dx = 2(x-1)\,dx$

$\frac{1}{2}\,du = (x-1)\,dx$

$\frac{3}{2}\int_{u(1)}^{u(5)} u^{1/2}\,du = \frac{3}{2}\cdot\frac{2}{3}u^{3/2} + C$

$\int_1^5 3(x-1)\sqrt{x^2-2x+10}\,dx = (x^2-2x+10)^{3/2}\Big|_1^5$

$= 25^{3/2} - 9^{3/2} = 98$

2. $F'(x) = \dfrac{12(x-2)}{(x+1)^{1/3}}$ $\quad F(0) = 8 \quad$ *Find* $F(x)$. Show all work.

$u = x+1 \qquad x = u-1 \qquad x-2 = u-3$

$du = dx$

$$\int \frac{12(u-3)}{u^{1/3}}\,du = \int (12u-36)\,u^{-1/3}\,du$$

distribute

$$\int 12u^{2/3} - 36u^{-1/3}\,du = 12\cdot\frac{3}{5}u^{5/3} - 36\cdot\frac{3}{2}u^{2/3} + C$$

$$= \frac{36}{5}(x+1)^{5/3} - 54(x+1)^{2/3} + C$$

Find $C$:

$\frac{36}{5} - 54 + C = 8$

$C = 8 + 54 - \frac{36}{5}$

$C = 44.8$

$$F(x) = \frac{36}{5}(x+1)^{5/3} - 54(x+1)^{2/3} + 44.8$$

3. Find the average of the left and right hand Riemann sums for $f(x) = 5 - 4x^2$ on the interval [ 0, 2 ] using 4 equal subintervals. Compare to $\int_0^2 f(x)dx$, rounded to 3 decimal places. Write the answers below.

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

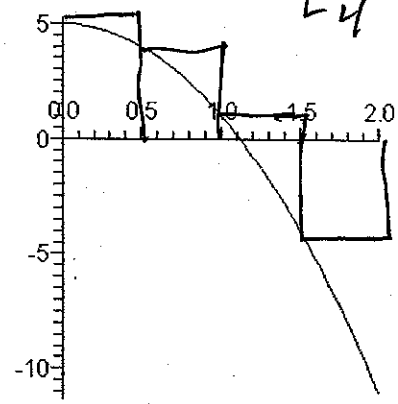
Find and sketch $L_4$ showing all work.

| $x$ | $5-4x^2$ |
|---|---|
| 0 | 5 |
| $\frac{1}{2}$ | 4 |
| 1 | 1 |
| $3/2$ | $-4$ |
| 2 | $-11$ |

$L_4 = \frac{1}{2}[5+4+1-4]$

$= 3$

$L_4$

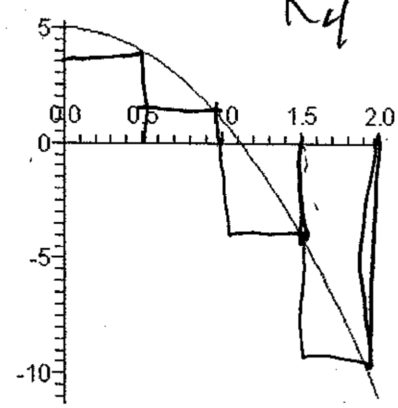



Find and sketch $R_4$ showing all work.

$R_4 = \frac{1}{2}[4+1-4-11]$

$= -5$

$R_4$




$\frac{L_4 + R_4}{2} =$ $\frac{3+(-5)}{2} = -1$

$\int_1^5 f(x)dx =$ $-2/3 = -.666\ldots$

You may use the calculator for the true value of the integral.