Math 142 Final Exam Review Solutions J Lewis
F2011

1. a) $\frac{\Delta p}{\Delta x} = \frac{-.5}{5} = -.1$ (30, 5) is a given p.t.

Pt. Slope $y - y_1 = m(x - x_1)$

$p - 5 = -.1(x - 30)$ $p - 5 = -.1x + 3$

Demand price → $\boxed{p = -.1x + 8}$

Fixed cost = 60 variable cost = $1 \cdot x = x$

$C(x) = x + 60$ $P(x) = x \cdot p - C(x)$ or $R(x) - C(x)$

$= -.1x^2 + 8x - (x + 60)$

$$\boxed{P(x) = -.1x^2 + 7x - 60}$$

b) $MP = P'(x) = -.2x + 7$

c) Solve $P'(x) = 0$ $-.2x + 7 = 0$ $7 = .2x$

$x = \frac{7}{.2} = 35$ units $\underset{\text{price}}{p} = -.1(35) + 8 = \$4.50$

2. a) $2^{x+2} = 5$

$(x+2)\ln 2 = \ln 5$ → $(x+2) = \frac{\ln 5}{\ln 2}$ $x = \frac{\ln 5}{\ln 2} - 2$

$x \approx .321928$

Note: log works just as well. We do not have $\log_2 x$ in the calculator or we could use it.

b) $\ln(5x-2) - \ln(x^2) = \ln 2$

$\ln\left(\frac{5x-2}{x^2}\right) = \ln 2$

$\frac{5x-2}{x^2} = 2$ → $5x - 2 = 2x^2$

$2x^2 - 5x + 2 = 0$

$(2x - 1)(x - 2) = 0$

$\boxed{x = \tfrac{1}{2} \text{ or } x = 2}$

Both make $5x-2$ and x pos. so both work in the original.

2 c) $\ln[(x+2)(x-2)] = 1$

$\ln(x^2-4) = 1$

$x^2 - 4 = e^1 = e$

$x^2 = e+4$

$\boxed{x = \sqrt{e+4}}$

We cannot use $x = -\sqrt{e+4}$ because then $x+2$ and $x-2$ would be negative.

3. a) $1000\left(1+\frac{.08}{4}\right)^{4\cdot 10} = 1000(1.02)^{40} = \$22080.40$

rdd. to nearest ¢

b) $10000e^{.08(10)} = 10000e^{.8} = \$22,255.41$

c) Solve $Pe^{.08t} = 2P$ $\quad e^{.08t} = 2$ $\quad .08t = \ln 2$

$t = \frac{\ln 2}{.08} \approx 8.66434$ yrs or about 8yrs 8mo.

4. Solve $Pe^{.07(10)} = 30,000$ for P.

$Pe^{.7} = 30,000$ $\quad P = \frac{30,000}{e^{.7}} \approx \$14897.56$

5. a) $x=-1$ and $x=1$ since the denominator is 0 there making f undefined.

b) Each piece is continuous so check for gaps at $x=-1$ and $x=3$.

$\lim_{x\to -1}(x^2+2x-5) = (-1)^2+2(-1)-5 = 1-7 = -6 = \lim_{x\to -1^-} f(x)$

$\lim_{x\to -1}(x-5) = -1-5 = -6 = \lim_{x\to -1^+} f(x) = f(-1)$

so f is continuous at $x=-1$.

$\lim_{x\to 3^-} f(x) = 3-5 = -2$ $\quad \lim_{x\to 3^+} f(x) = 5-9 = -4$

so f is not continuous at $x=3$.

$\boxed{x = 3 \text{ only}}$

6. a) Graph $f(x)$ in a window such as $[-1,1] \times [-1,1]$, you see a cusp at $x=0$.

Otherwise find $f'(x) = \frac{2}{3}x^{-1/3}$ is undefined at $x=0$. f is not differentiable at $\boxed{x=0.}$

b) Each piece is differentiable on its interval.

Check for i) discontinuity ii) c

at $x=1$ and $x=3$

$x=1$: i) $1^2-1=0=\lim_{x\to1^-} f(x) = f(1) = 1-1 = 0 = \lim_{x\to1^+} f(x)$ ✓

ii) Is there a corner at $x=1$?

The slope of x^2-x at $x=1$ is $(2x-1)\Big|_{x=1} = 2-1 = 1$

The slope of $x-1$ at $x=1$ is 1

These match, no corner

$x=3$: i) $\lim_{x\to3^-} f(x) = 3-1 = 2 = f(3)$ $\lim_{x\to2^+} f(x) = 9-15+8 = 2$ ✓

ii) Is there a corner at $x=3$?

The slope of $x-1$ is 1.

The slope of x^2-5x+8 at $x=3$ is $(2x-5)\Big|_{x=3} = 6-5 = 1$

match No corner at $x=3$

$\boxed{f(x) \text{ is differentiable for all } x.}$

7 a) $f(x) = xe^{0.5x}$

Product Rule
$(fg)' = f'g + fg'$

$f'(x) = e^{0.5x} + .5xe^{0.5x}$

$= e^{0.5x}(1 + .5x)$

Tangent is horizontal when $f'(x) = 0$.
$e^{0.5x}$ is never 0 so solve $1 + .5x = 0$ $\boxed{x = -2}$

b) $f(x) = \frac{\ln x}{x}$

Quotient Rule
$\left(\frac{T}{B}\right)' = \frac{T'B - TB'}{B^2}$

$f'(x) = \frac{\frac{1}{x} \cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$

Solve $1 - \ln x = 0$ $\quad 1 = \ln x$ $\quad \boxed{x = e}$

c) $f(x) = (x^2+1)^3(x^2-1)^4$

Product Rule and Chain Rule

$f'(x) = 3(x^2+1)^2 \cdot 2x\,(x^2-1)^4 + (x^2+1)^3 \cdot 4(x^2-1)^3 \cdot 2x$

Factor out $2x(x^2+1)^2(x^2-1)^3$ to get

$2x(x^2+1)^2(x^2-1)^3\left[3(x^2-1) + 4(x^2+1)\right]$

$= 2x\underbrace{(x^2+1)^2}_{\geq 1 \text{ for all } x}(x^2-1)^3\underbrace{\left[7x^2 + 1\right]}_{7x^2+1 \geq 1 \text{ for all } x}$

Only $2x(x^2-1)^3$ can be non postive.

The tangent line is horizontal at
$x = 0$, $x = -1$, $x = 1$

7. d) $f'(x) = \sqrt{x^2+1} + x \cdot \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x$

$$= \sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}}$$

Both are nonnegative and $\sqrt{x^2+1} \geq 1$ for all x so the tangent line is never horizontal.

e) Rewriting $f(x)$:

$$f(x) = \ln(x^2+1) + \ln(x-3) - \ln(x+2)$$

$$f'(x) = \frac{2x}{x^2+1} + \frac{1}{x-3} - \frac{1}{x+2}$$

f) $f(x) = 3^{x^2+1} \cdot (2x) \ln 3$ T.L. is horizontal at $x=0$.

8. $y = f'(a)(x-a) + f(a)$

a) $f'(x) = \frac{1}{2}(x^2+16)^{-1/2}(2x) = \frac{x}{\sqrt{x^2+16}}$

$f'(3) = \frac{3}{\sqrt{9+16}} = \frac{3}{5} = \text{slope}$

$f(3) = \sqrt{9+16} = 5$ T.L.: $\boxed{y = \frac{3}{5}(x-3)+5}$

8 b) $f(x) = 3e^{x+2}$ $\quad a = -2$

$f'(x) = 3e^{x+2}$

$f'(-2) = 3$ $\quad f(-2) = 3$

T.L.: $\quad y = 3(x+2) + 3$

9. a) $p = 300 - \frac{1}{2}x$ $\quad R(x) = 300x - \frac{1}{2}x^2$

b) $P(x) = R(x) - C(x)$ $\quad \underline{C(x) = 120x + 10000}$

$P(x) = -\frac{1}{2}x^2 + 180x - 10000$

c) $P'(x) = -x + 180$. so P_{max} is at $x = 180$.

d) $P'(160) = 20$ dollars/unit

e) $y = 20(x - 160) + 6000$
$(P(160) = 6000)$

f) $C(x) = 120x + 10000$

$AC = \frac{C(x)}{x} = 120 + \frac{10000}{x}$

$MAC = \left(\frac{C(x)}{x}\right)' = -\frac{10000}{x^2}$

10. a) $f(x) = \dfrac{2x+3}{3x-6}$

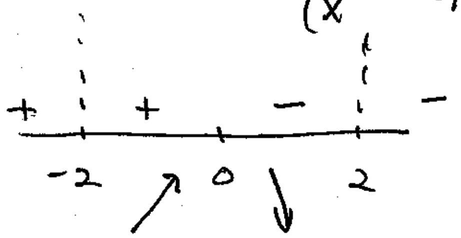
$$f'(x) = \frac{2(3x-6) - (2x+3)\cdot 3}{(3x-6)^2}$$

$$= \frac{-3}{(3x-6)^2}$$ no extrema or inflection pts

$$f''(x) = \frac{-18}{(3x-6)^3}$$

b) $$f'(x) = \frac{4x(x^2-4) - 2x(2x^2+1)}{(x^2-4)^2}$$

$$= \frac{2x\,(2x^2-8-2x^2-1)}{(x^2-4)^2} = \frac{-9x}{(x^2-4)^2}$$



$\boxed{\text{relative max at } x=0}$

$$f''(x) = \frac{-9(x^2-4)^2 + 9x\cdot 2(x^2-4)\cdot 2x}{(x^2-4)^4} = (x^2-4)[\quad]$$

$$= \frac{-9x^2+36+36x^2}{(x^2-4)^3}$$

$$= \frac{27x^2+36}{(x^2-4)^3}$$ is never 0, only undefined at the V.A's of f so no inflection pts.

f does change concavity however at -2 and 2.

c) $f(x) = xe^{0.5x}$

$$f'(x) = e^{0.5x} + .5xe^{0.5x} = e^{0.5x}(1+.5x)$$

$f'(x) = 0$ at $x = -2$.

f': $-$ | $+$ (at -2)

f: $\searrow$ -2 $\nearrow$

$\boxed{\text{relative min. at } x=-2}$

$\boxed{(-2, -2e^{-1}) \text{ is the min pt.}}$

$$f''(x) = [e^{0.5x}(1+.5x)]'$$
$$= .5e^{0.5x} + e^{0.5x}(.5)(1+.5x)$$
$$= e^{.5x}[.5 + .5 + .25x]$$
$$= e^{.5x}(1+.25x) \quad = 0 \text{ at } x = -4$$

f'': $-$ | $+$ (at -4)

f: $\frown$ -4 $\smile$

$\boxed{\text{I.P. at } (-4, -4e^{-2})}$

d) $f(x) = \dfrac{e^x}{x}$ $\quad f'(x) = \dfrac{xe^x - e^x}{x^2} = \dfrac{(x-1)e^x}{x^2}$

$-$ | $-$ | $+$ (at 0, 1)

0 $\searrow$ 1 $\nearrow$

$\boxed{\text{relative min at } (1, e)}$

10 d continued

$$f''(x) = \frac{e^x + (x-1)e^x - 2x(x-1)e^x}{x^4}$$

$$= e^x\left[1 + x - 1 - 2x(x-1)\right] / x^4$$

$$= e^x(x - 2x^2 + 2x)/x^4$$

$$= e^x\underbrace{(3x - 2x^2)}_{x(3-2x)}/x^4$$

$$= e^x(3-2x)/x^3$$

$= 0$ at $x = \frac{3}{2}$

undefined at $x = 0$

f'': $-$ | 0 | $+$ | $\frac{3}{2}$ | $-$

f: $\cap$ | 0 VA | $\cup$ | $\frac{3}{2}$ IP | $\cap$

$\boxed{\text{I.P. at } x = \frac{3}{2}}$

e) $f(x) = \dfrac{\ln x}{x}$ Domain $= (0, \infty)$

$$f'(x) = \frac{\frac{1}{x}\cdot x - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$= 0$ at $x = e$

undef. at $x = 0$ VA

f': 0 | $+$ | e | $-$

f: $\nearrow$ | $\searrow$

$\boxed{\text{relative max at } \left(e, \frac{1}{e}\right)}$

$$f''(x) = \frac{-\frac{1}{x}\cdot x^2 - (1 - \ln x)\cdot 2x}{x^4} = \frac{-x - 2x(1 - \ln x)}{x^4}$$

$$= \frac{-1 - 2(1 - \ln x)}{x^3} = \frac{1 + 2\ln x}{x^3}$$

$= 0$ at $\ln x = -\frac{1}{2}$

$x = e^{-1/2}$

undef. at $x = 0$

10 e) continued

f'': $-$ | $+$ on $(0, \infty)$, split at $e^{-1/2}$

f: $\frown$ IP $\smile$

$\boxed{\text{I. P. at } \left(e^{-1/2}, \frac{-\sqrt{e}}{2}\right)}$

f) $f(x) = x \ln x$ Domain $= (0, \infty)$

$f'(x) = \ln x + x \cdot \frac{1}{x} = \ln x + 1 = 0$

$\ln x = -1$

$x = e^{-1} = 1/e$

f': $-$ | $+$ on $(0, \infty)$, split at $1/e$

f: $\searrow$ $1/e$ $\nearrow$

$\boxed{\text{relative min at } (1/e, -1/e)}$

$f''(x) = \frac{1}{x}$

$\boxed{\text{No inflection pts.}}$

11. a) $\int x^{4/5 - 2} - e^x \, dx = \int x^{-6/5} - e^x \, dx$

$= 5x^{1/5} - e^x + C$

b) $\int \frac{x^2}{x^3} + x^{-3} \, dx = \int \frac{1}{x} + x^{-3} \, dx$

$= \ln|x| - \frac{1}{2} x^{-2} + C = \ln|x| - \frac{1}{2x^2} + C$

c) $u = x-3$ $\quad x = u+3$

$du = dx$ $\quad x+1 = u+4$

$$\int (u+4)u^{1/2}\,du = \int u^{3/2} + 4u^{1/2}\,du$$

$$= \frac{2}{5}u^{5/2} + \frac{8}{3}u^{3/2} + C = \frac{2}{5}(x-3)^{5/2} + \frac{8}{3}(x-3)^{3/2} + C$$

d) $u = 1+2x^3$

$du = 6x^2\,dx$

$\frac{1}{6}\,du = x^2\,dx$

$$\int \frac{5/6\,du}{u} = \frac{5}{6}\ln|u| + C$$

$$= \frac{5}{6}\ln|1+2x^3| + C$$

e) $u = x^{3/2}$

$du = \frac{3}{2}x^{1/2}\,dx$

$\frac{2}{3}du = \sqrt{x}\,dx$

$$\int \frac{2}{3}e^u\,du = \frac{2}{3}e^u + C$$

$$= \frac{2}{3}e^{x^{3/2}} + C$$

f) $u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$2\int e^u\,du$$

$$= 2e^u + C$$

$$= 2e^{\sqrt{x}} + C$$

g) $u = x^3 - 6x^2 + 1$

$du = (3x^2 - 12x)\,dx$

$= 3(x^2 - 4x)\,dx$

$\frac{1}{3}du = (x^2 - 4x)\,dx$

$\frac{1}{3}\int e^u\,du$

$= \frac{1}{3}e^u + C$

$= \frac{1}{3}e^{(x^3 - 6x^2 + 1)} + C$

h) $u = \ln x$

$du = \frac{1}{x}\,dx$

$\int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + C$

$= \frac{2}{3}(\ln x)^{3/2} + C$

i) $u = x + 1$ $\quad x = u - 1$ $\quad \int (u-1)^2 u^4\,du$

$du = dx$

$= \int (u^2 - 2u + 1)u^4\,du = \int u^6 - 2u^5 + u^4\,du$

$= \frac{1}{7}(x+1)^7 - \frac{1}{3}(x+1)^6 + \frac{1}{5}(x+1)^5 + C$

12. $\left(\frac{C(x)}{x}\right)' = \frac{-25000}{x^2}$

$\frac{C(x)}{x} = \frac{25000}{x} + C$

$\frac{25000}{1000} + C = 65$

$25 + C = 65$

$C = 40$

a) $\frac{C(x)}{x} = \frac{25000}{x} + 40$

b) $C(x) = 25000 + 40x$

c) $\frac{1}{1000}\int_0^{1000}(25000 + 40x)\,dx = \$45000$

13. a) $f(x) = \ln|x| + C$ $\quad \ln e + C = 2 \quad 1 + C = 2$

$C = 1$

$\boxed{f(x) = \ln|x| + 1}$

b) $f'(x) = \frac{1}{2} + \frac{3}{2x}$

$f(x) = \frac{1}{2}x + \frac{3}{2}\ln|x| + C$

$f(1) = \frac{1}{2} + 0 + C = 2.5 \quad C = 2$

$\boxed{f(x) = \frac{1}{2}x + \frac{3}{2}\ln|x| + 2}$

14. $\Delta x = \frac{3-0}{6} = \frac{1}{2}$

x	$f(x)$
0	1.
$\frac{1}{2}$	1.25
1	2
$\frac{3}{2}$	3.25
2	5
$5/2$	7.25
3	10

a) $L_6 = \frac{1}{2}[19.75]$

$= 10.25$

b) $R_6 = \frac{1}{2}[28.75]$

$= 14.375$

c) $\int_0^3 1+x^2\,dx = x + \frac{1}{3}x^3\Big|_0^3 = 3 + \frac{27}{3} - 0 = 12$

$L_6 < \int_0^3 1+x^2\,dx < R_6$ in this case.

Note. $\frac{L_6+R_6}{2} = \frac{24.625}{2} = 12.3125$

15, a) Math 9 Math NUM(enter x^2+x-6); x, -3, 4)

$x^2+6x-2-(5x+4)$
$= x^2+x-6$

$= \boxed{33.5}$

b) Math 9 Math Num(enter x^3-8), x, -2, 5)

$x^3+2x-1-(2x+7)$
$= x^3-8$

$= \boxed{160.25}$

16. $Y1 = 100 - 0.2x$ $\quad 75 = 0.25x$ $\quad 100 - .2(300)$

$Y2 = 25 + 0.05x$ $\quad 300 = \bar{x}$ $\quad \bar{p} = 100 - 60 = 40$

or 2nd trace 5 enter 3 times $Eq = (300, 40)$

$$C.S. = \int_0^{300} Y1 - 40 \, dx = \$9000$$

$$P.S. = \int_0^{300} 40 - Y2 \, dx = \$2250$$

17. $f(x,y) = x^2 e^{xy}$

$$\frac{\partial f}{\partial x} = 2x e^{xy} + x^2 y e^{xy} \qquad \frac{\partial f}{\partial y} = x^3 e^{xy}$$

$\frac{\partial f}{\partial y} = 0$ if and only if $x = 0$.

$$\frac{\partial f}{\partial x}(0,y) = 0 + 0 = 0$$

All pts. $(0, y)$ are critical pts.

18. a) $\frac{\partial f}{\partial x} = 3y - 3x^2 \qquad \frac{\partial f}{\partial y} = 3x - 3y^2$

$\frac{\partial f}{\partial x} = 0$ iff $y = x^2 \rightarrow \frac{\partial f}{\partial y}(x, x^2) = 3x - 3x^4 = 0$

$3x(1 - x^3)$

$x = 0$ or $x = 1$

$$\boxed{\begin{array}{l}(0,0) \quad (1,1) \\ \text{are critical pts.}\end{array}}$$

18 a) continued

$$\frac{\partial^2 f}{\partial x^2} = -6x \qquad \frac{\partial^2 f}{\partial y^2} = -6y \qquad \frac{\partial^2 f}{\partial x \partial y} = 3$$

$D(x,y) = (-6x)(-6y) - 3^2 = 36xy - 9$

$D(0,0) = 0 - 9 < 0$ $\boxed{\text{saddle pt at } (0,0,0)}$

$D(1,1) = 36 - 9 > 0$

$\frac{\partial f}{\partial x}(1,1) = -6 < 0$ $\boxed{\text{relative max at } (1,1,1)}$

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b) $f(x,y) = x^2y - 2y^2 + 4xy$

$\frac{\partial f}{\partial x} = 2xy + 4y \qquad \frac{\partial f}{\partial y} = x^2 - 4y + 4x$

$\frac{\partial f}{\partial x} = 2y(x+2)$
$= 0$ iff
$y=0$ or $x=-2$

case1: $y=0$

$\frac{\partial f}{\partial y}(x,0) = x^2 + 4x$
$= x(x+4)$
$= 0$ if $x=0$ or $x=-4$

$\boxed{(0,0) \quad (0,-4)}$ are two c.P.'s

18 b) continued

Case 2: $x = -2$

$$\frac{\partial f}{\partial y}(-2, y) = 4 - 4y - 8 = -4y - 4 = 0$$

if $y = -1$

$\boxed{(-2,-1) \text{ is another c.p.}}$

3 c.p.'s are $(0,0)$ $(0,-4)$ and $(-2,-1)$.

Classify each:

$$\frac{\partial^2 f}{\partial x^2} = 2y \qquad \frac{\partial^2 f}{\partial y^2} = -4 \qquad \frac{\partial^2 f}{\partial x \partial y} = 2x+4$$

$$D(x,y) = 2y(-4) - (2x+4)^2$$

$$D(0,0) = 0 - 16 < 0$$

$\boxed{\text{saddle pt at } (0,0,0)}$

$$D(0,-4) = 32 - 16 = 16 > 0$$

$$\frac{\partial^2 f}{\partial y^2} = -4 < 0$$

$\boxed{\text{relative max at } (0,-4,-16)}$

$$D(-2,-1) = 8 - (0)^2 = 8 > 0$$

$$\frac{\partial^2 f}{\partial y^2} = -4 < 0$$

$\boxed{\text{relative max at } (-2,-1,2)}$

18c) $f(x,y) = x^2y^2 - 2x^4 - 4y^2$

$$\frac{\partial f}{\partial x} = 2xy^2 - 8x^3 \qquad \frac{\partial f}{\partial y} = 2x^2y - 8y$$

$$= 2y(x^2-4) = 0$$

iff $y=0$, $x=-2$

or $x=2$.

Case 1: $y=0$

$$\frac{\partial f}{\partial x}(x,0) = -8x^3 = 0 \text{ iff } x=0$$

C.P. $(0,0)$

Case 2: $x=-2$

$$\frac{\partial f}{\partial x}(-2,y) = -4y^2 - 8(-8) = -4y^2 + 64 = 0$$

iff $y^2 = 16$ $\quad y = \pm 4$

C.P.'s
$(-2,-4)$
$(-2,4)$

Case 3: $x=2$

$$\frac{\partial f}{\partial x}(2,y) = 4y^2 -$$

again $4y^2 = 64$ $\quad y = \pm 4$

C.P.'s
$(2,-4)$
$(2,4)$

The 3 C.P.'s are $\boxed{(0,0), (2,-4) \text{ and } (2,4).}$

To classify, find $D(x,y)$.

$$\frac{\partial^2 f}{\partial x^2} = 2y^2 - 24x^2 \quad \frac{\partial^2 f}{\partial y^2} = 2x^2 - 8 \quad \frac{\partial^2 f}{\partial x \partial y} = 4xy$$

$$D(x,y) = (2y^2 - 24x^2)(2x^2 - 8) - (4xy)^2$$

$$D(0,0) = 0 - 0 = 0$$

$\boxed{\text{Test fails at } (0,0).}$

$$D(2,-4) = (16 - 96)(0) - (4(-8))^2$$
$$= -(16(64)) < 0$$

$\boxed{\text{saddle pt at } (2,-4)}$

$$D(2,4) = 0 - 16 \cdot 64 < 0$$

$\boxed{\text{saddle pt at } (2,4)}$

J Lewis

Optimization Final Review Solutions

1. Let w = number of new workers

profit/unit $= 40.50 - 3w$

#units $= 15 + 2w$

Profit = profit/unit × # units

$P(w) = (40.50 - 3w)(15 + 2w)$

$P'(w) = -3(15+2w) + (40.50 - 3w)\cdot 2$

$0 = -45 - 6w + 81 - 6w = 36 - 12w$

$36 = 12w \qquad \boxed{w = 3}$

$P''(w) = -12 < 0$ shows $w=3$ is really a max. pt.

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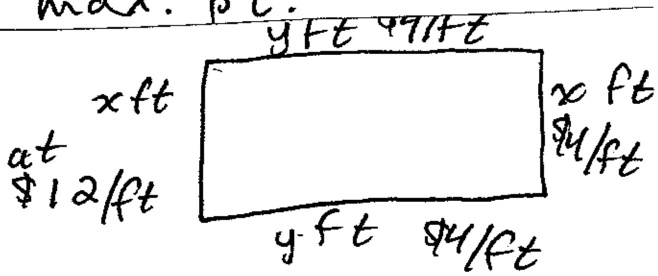
2. $C = 900 = 12x + 4x + 8y$

$C = 16x + 8y = 900$



$A = xy = x\left(\dfrac{900 - 16x}{8}\right) = x\left(\dfrac{900}{8} - 2x\right)$

$A = \dfrac{900}{8}x - 2x^2$

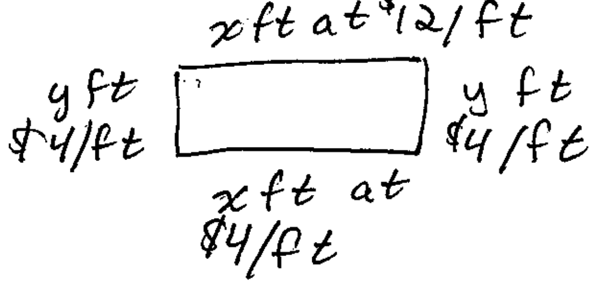
$A'(x) = \dfrac{900}{8} - 4x = 0$ if $x = \dfrac{900}{32}$

$\boxed{x = 28.125}$

$\boxed{y = \dfrac{900 - 16(28.125)}{8} = 56.25}$

3. $A = xy = 1296$

$y = \dfrac{1296}{x}$



$$C = 16x + 8y = 16x + \frac{8(1296)}{x}$$

$$C'(x) = 16 - \frac{8(1296)}{x^2} = 0 \text{ if } x^2 = \frac{8(1296)}{16}$$

$$16 = \frac{8(1296)}{x^2}$$

$$16x^2 = 8(1296)$$

$$x^2 = \frac{1296}{2}$$

$$x^2 = 648$$

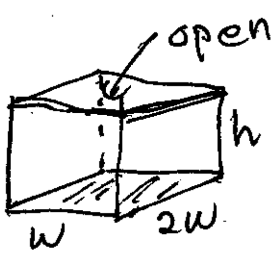
$$\boxed{x \approx 25.456}$$

$$y = \frac{1296}{\sqrt{648}}$$

$$= 2\sqrt{648}$$

$$\boxed{y \approx 50.912}$$

4.



$V = 2w^2 h = 972 \qquad h = \dfrac{972}{2w^2}$

Surface area $= S$

$$S = \underset{\text{front}}{wh} + \underset{\text{back}}{wh} + \underbrace{2wh + 2wh}_{\text{sides}} + \underset{\text{base}}{2w^2}$$

$$S = 6wh + 2w^2 = \frac{6(972)}{2w^2} + 2w^2$$

$$= \frac{3(972)}{w} + 2w^2$$

$$S'(w) = -\frac{3(972)}{w^2} + 4w = 0$$

if $\dfrac{3(972)}{w^2} = 4w \qquad w^3 = \dfrac{3(972)}{4}$

$$w = \sqrt[3]{729} = 9$$

$$\boxed{\begin{matrix} w & l & h \\ 9 \times & 18 \times & 6 \end{matrix}}$$