Math 142 Final Review

1. A company finds the daily demand for its cookies is a linear function of price. If price increases by \$0.50 per lb, demand decreases by 5 lbs. The daily demand when price is \$5 per lb is 30 lbs. It costs them \$60 per day for labor and an additional \$1.00 per lb for ingredients.

a) Find the profit function.
b) Find the marginal profit.
c) Find the quantity and price for maximum profit.

2. Solve for x:

a) $2^{x+2} = 5$ b) ln(5x-2) – 2ln(x)=ln2 c) ln(x+2) + ln(x-2)=1

3. Find the amount after 10 years if \$10,000 is invested at 8% compounded
a) quarterly b) continuously c) How long will it take the amount to double if compounded continuously?

4. Find the present value of \$30,000 ten years from now if money earns 7% compounded continuously.

5. Find all discontinuities.

a) $f(x) = \dfrac{x^2 - 3x - 4}{x^2 - 1}$ b) $f(x) = \begin{cases} x^2 + 2x - 5 & x < -1 \\ x - 5 & -1 \le x < 3 \\ 5 - x^2 & 3 \le x \end{cases}$

6. Find the points where f is not differentiable.

a) $f(x) = x^{\frac{2}{3}}$ b) $f(x) = \begin{cases} x^2 - x & x \le 1 \\ x - 1 & 1 < x \le 3 \\ x^2 - 5x + 8 & 3 < x \end{cases}$

7. Find the derivatives and determine where the tangent line is horizontal.

a) $f(x) = xe^{0.5x}$ b) $f(x) = \dfrac{\ln(x)}{x}$ c) $f(x) = (x^2+1)^3(x^2-1)^4$

d) $f(x) = x\sqrt{x^2+1}$ e) $f(x) = \ln\left[\dfrac{(x^2+1)(x-3)}{x+2}\right]$ Use log rules. Do not solve for horizontal tangent line.

f) $f(x) = 3^{x^2+1}$

8. Find the equation of the tangent line at x=a.

a) $f(x) = \sqrt{x^2 + 16}$ a=3 b) $f(x) = 3e^{x+2}$ a=-2

9. Demand for a commodity is x=600-2p. The cost equation for the same commodity is C(x)=120x+10,000.

a) Find the revenue as a function of x.
b) Find the profit function.
c) What is the maximum profit and at what quantity does it occur?
d) Approximate the additional profit if the quantity produced and sold increases from 160 to 161. Use the marginal profit.
e) Find the equation of the tangent line at x=160.
f) Find the marginal average cost at x=160.

10. Find all local extrema and inflection points of each.

a) $f(x) = \dfrac{2x+3}{3x-6}$ b) $f(x) = \dfrac{2x^2+1}{x^2-4}$ c) $f(x) = xe^{0.5x}$ d) $f(x) = \dfrac{e^x}{x}$

e) $f(x) = \dfrac{\ln(x)}{x}$ f) f(x)= xln(x)

11. Find the antiderivatives:

a) $\int \dfrac{x^{\frac{4}{5}} - x^2 e^x}{x^2} dx$ b) $\int \dfrac{x^2+1}{x^3} dx$ c) $\int (x+1)\sqrt{x-3}\,dx$ d) $\int \dfrac{5x^2}{1+2x^3} dx$

e) $\int \sqrt{x} e^{x^{\frac{3}{2}}} dx$ f) $\int \dfrac{e^{\sqrt{x}}}{\sqrt{x}} dx$ g) $\int (x^2 - 4x)e^{x^3-6x^2+1} dx$ h) $\int \dfrac{\sqrt{\ln(x)}}{x} dx$

i) $\int x^2 (x+1)^4 dx$

12. A marginal average cost function is $MAC = \dfrac{-25000}{x^2}$. The average cost per unit for producing 1000 units is $65 per unit.

a) Find the average cost per unit function.
b) Find the cost function.
c) Find the average value of the cost function for x between 0 and 1000.

13. Find the antiderivative that satisfies the given condition.

a) $\frac{df}{dx} = \frac{1}{x}$ $f(e)=2$ b) $\frac{df}{dx} = \frac{x+3}{2x}$ $f(1)=2.5$

14. Find the Riemann sums for $f(x) = 1 + x^2$, using 6 equal intervals on [0,3]

a) and the left endpoint of each interval.
b) and the right endpoint of each interval.
c) How do these compare to

$$\int_0^3 1 + x^2 dx$$

15. Find the area between the curves over the given interval.

a) $f(x) = x^2 + 6x - 2$ $g(x) = 5x + 4$ [-3, 4]

b) $f(x) = x^3 + 2x - 1$ $g(x) = 2x + 7$ [-2, 5]

16. Find the consumer's surplus and the producer's surplus at equilibrium if the demand price is D(x)= 100 – 0.2x and the supply price is S(x)= 25 + 0.05x.

17. Find the critical points of $f(x, y) = x^2 e^{xy}$

18. Find the critical point(s) and classify by the 2nd derivative test if possible.

$a)$ $f(x, y) = 3xy - x^3 - y^3$ $b)$ $f(x, y) = x^2 y - 2y^2 + 4xy$ $c)$