Math 142 ICE 1 Rev. Solutions F2011

1. Let t = baby's age in months.
For the first 4 months
$W(t) = mt + b$ since W is linear.
$m = 2$ is given, $b = 8$ is given.
$W(t) = 2t + 8 \quad t \le 4$

$W(t)$ for $4 \le t \le 12$ is linear.
$W(4) = 2(4) + 8 = 16$ and
$W(12) = 24$ is given. The slope from
$(4, 16)$ to $(12, 24)$ is $\frac{24-16}{12-4} = 1$

so $W(t) = 1(t-4) + 16$ for $4 \le t \le 12$
(pt. slope)

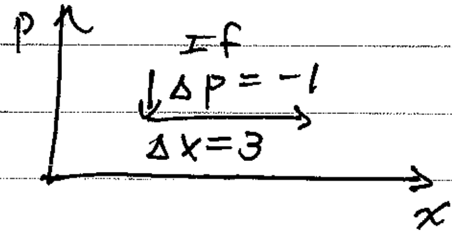
$$W(t) = \begin{cases} 2t+8 & t \le 4 \\ t+12 & 4 < t \le 12 \end{cases}$$

2. $C(x) = 21x + 120$

Find the demand equation for

x = quantity p = price/unit

Given $(12, 58)$ and $m = -\frac{1}{3}$



so $m = -1/3$

Pt. Slope : $y - 58 = -\frac{1}{3}(x - 12)$

or $p = -\frac{1}{3}x + 62$

solve p=0 or guess

Then $R(x) = xp = -\frac{1}{3}x^2 + 62x$

Profit is

$P(x) = -\frac{1}{3}x^2 + 62x - (21x + 120)$

$= -\frac{1}{3}x^2 + 41x - 120$

Break Even at $x = 3$ and $x = 120$

A good window has $X_{MIN} = 0$

$X_{MAX} = \frac{62}{1/3} = 186$

$Y_{min} = 0$

Y_{max} guess

3. $Pe^{.064t} = 2P \quad e^{.064t} = 2$

$$.064t = \ln 2$$
$$t = \frac{\ln 2}{.064} \approx 10.83 \text{ years}$$

4. $Pe^{2r} = 1525.63 \quad Pe^{3r} = 1575.04$

$$\frac{Pe^{3r}}{Pe^{2r}} = e^{r} = \frac{1575.04}{1525.63}$$
$$r = \ln\left(\frac{1575.04}{1525.63}\right) \cong \boxed{.0318732293} = r$$
$$P = \frac{1525.63}{e^{2r}} = \frac{1525.63}{e^{.0637464586}}$$
$$\approx \boxed{\$1431.41} \approx P$$

5. $500{,}000e^{.02(2)} = 520405 3.871$

or 5204054 people

6. a) $\log_4(x^2-9) = 2 \quad x^2-9 = 4^2 = 16$

$$x^2 - 25 = 0$$
$$x = -5 \text{ or } x = 5$$

but -5 makes $\log_4(x-3)$ and $\log_4(x+3)$

undefined so only $\boxed{x=5}$

6b) $\log x^4 + \log\sqrt{x^2+1} - \log(x+2)^3$

$= \log x^4\sqrt{x^2+1} - \log(x+2)^3$

$= \log\left[\dfrac{x^4\sqrt{x^2+1}}{(x+2)^3}\right]$

7. a) Substitition of $x=-3$ gives $\frac{0}{20}=0$
$\lim_{x\to -3} g(x) = 0$.

b) Substitution of $x=-1$ gives $\frac{-18}{0}$
so g has a vertical asymptote $x=-1$.
Observe in the calculator,
$\lim_{x\to -1^-} g(x) = -\infty$

c) $\lim_{x\to -1^+} g(x) = +\infty$ by observing graph.

d) The graph looks continuous in the calculator but the denominator of $g(x)$ is 0 at $x=2$. So g has a hole at $x=2$. Look in table or factor and cancel.

$$g(x) = \frac{3(x^2+x-6)}{2(x^2-x-2)} = \frac{3}{2}\,\frac{(x+3)\cancel{(x-2)}}{(x+1)\cancel{(x-2)}} \quad x \neq 2$$

$$\lim_{x\to 2} g(x) = \frac{3}{2}\cdot\frac{2+3}{2+1} = \frac{15}{6} = \frac{5}{2} = 2.5$$

e) $\lim_{x\to\infty} g(x) = \frac{3}{2}$ by looking at the ratio of the highest degree terms.

8. Factor the denominator of the left piece.

$x^2+8x+16 = (x+4)^2$ is 0 if $x=-4$.

Since we use this formula at $x=-4$ and on each side of -4, f is not continuous at $x=-4$.
To see what fails at -4, we factor the numerator:

$$x^2-x-20 = (x+4)(x-5)$$

so $\frac{x^2-x-20}{x^2+8x+16} = \frac{(x+4)(x-5)}{(x+4)^2} = \frac{x-5}{x+4}$

There is a vertical asymptote at $x=-4$
$\lim_{x\to -4} f(x)$ DNE.

Now check $x=5$:

Does $\lim\limits_{x\to 5^-}\left(\frac{x^2-x-20}{x^2+8x+16}\right) = \lim\limits_{x\to 5^+} \ln(x-4)$?

or $\lim\limits_{x\to 5^-} f(x) = \frac{5-5}{5+4}$ $\left(\text{plugging 5 into } \frac{x-5}{x+4}, \text{ the reduced version}\right)$

$= \frac{0}{9} = 0$

$\lim\limits_{x\to 5^+} f(x) = \ln(5-4) = \ln 1 = 0$

and $f(5) = \ln(5-4) = 0$

f is continuous at $x=5$.

Since $x-4>0$ for $x \geq 5$, there are no more problem points.

Answer

$x=-4$ only V.A. $\lim\limits_{x\to -4} f(x)$ DNE

9. Look in the calculator.

9. a) Explanation way:

$$\frac{2e^x+7}{3e^x-9} = \frac{2+\cancel{7e^{-x}}\to 0}{3-\cancel{9e^{-x}}\to 0} \xrightarrow[x\to\infty]{} \frac{2}{3}$$

since $e^{-x} \xrightarrow[x\to\infty]{} 0$.

b) $$\frac{2e^x+7}{3e^x-9} \xrightarrow[x\to-\infty]{} \frac{7}{-9} \quad \text{since } e^x \xrightarrow[x\to-\infty]{} 0$$

c) $$\sim \frac{3x^4}{4x} \sim \frac{3}{4}x^3 \xrightarrow[x\to-\infty]{} -\infty$$

d) $$\sim \frac{2x^3}{4x^4} = \frac{1}{2x} \xrightarrow[x\to\infty]{} 0$$

10. First check to see if f is continuous. Each piece is a continuous function so we only need to look for gaps at the definition changing pts.

$$\lim_{x\to 1^-} f(x) = 1^{2/3} = 1 \qquad \lim_{x\to 1^+} f(x) = 1^2-6+6 = 1$$

and $f(1) = 1$ $\quad$ f is continuous at $x=1$.

11. Since $f(3) = y(3)$ and $f'(3) = y'(3) = \text{slope}$,

$f(3) = -2 \cdot 3 + 1 = -5$, $f'(3) = -2$

12.

a) Find the point of tangency:

$f(8) = 6 \cdot 8^{1/3} - 2 \cdot 8 + 1 = -3$

Find the slope $= f'(8)$.

$f'(x) = 6 \cdot \frac{1}{3} x^{-2/3} - 2$

$f'(8) = 6 \cdot \frac{1}{3}(8^{-2/3}) - 2$

$= 2\left(\frac{1}{(\sqrt[3]{8})^2}\right) - 2$ $\qquad \sqrt[3]{8} = 2$

$= \frac{1}{2} - 2$ $\qquad \frac{1}{2^2} = \frac{1}{4}$

$= -1.5$

Use Pt. Slope $(8, -3)$ $m = -1.5$

$$\boxed{y = -1.5(x - 8) - 3}$$

$g(4) = 1$ Tangency pt. is $(4, 1)$

12. b) Write $g(x)$ as a linear combination of power functions and use the power rule.

$$g(x) = \frac{x^2 - 8x + 16}{x^{\frac{1}{2}}} + 1$$

$$g(x) = x^{3/2} - 8x^{1/2} + 16x^{-1/2} + 1$$

$$g'(x) = \frac{3}{2}x^{1/2} - 4x^{-1/2} - 8x^{-3/2}$$

$$g'(4) = \frac{3}{2}\cdot 4^{1/2} - 4 \cdot 4^{-1/2} - 8 \cdot 4^{-3/2}$$

$$= 3 - 2 - 1 = 0$$

The tangent line is $y = 0(x-4) + 1$

or $\boxed{y = 1}$

a horizontal line

Math 142 In Class Exam 1 Review

1. A baby weighed 8 pounds at birth. His weight increased linearly so that he gained 2 pounds per month for the first 4 months. then he gained more slowly so that he weighed 24 pounds at 12 months of age. Write the piecewise definition of his weight as a function of t = his age in months.

2. A company making a single product has fixed costs totaling $120 per day. Each unit costs an additional $21 to produce. Demand quantity is 12 per day if the selling price is $58 and demand increases by 3 for each $1 decrease in the price. Find the cost, revenue and profit functions and find the break even quantities.

3. If $P is invested at 6.4%, how long will it take to double the value in the account if interest is compounded continuously?

4. $P is invested at annual interest rate, r, compounded continuously. The amount after 2 years is $1525.63 and the amount after 3 years is 1575.04. Find P and r.

5. A population grows at continuous annual rate 2%. If initially there were 500000 people, how many are there at 2 years?

6. a) Solve for x. $\log_4(x+3) + \log_4(x-3) = 2$

b) Write as a single logarithm. $4\log x + \frac{1}{2}\log(x^2+1) - 3\log(x+2)$

7. $g(x) = \dfrac{3x^2 + 3x - 18}{2x^2 - 2x - 4}$

Evaluate each limit as a number, infinity or minus infinity.

$a)\ \lim_{x\to -3} g(x)$ $\quad b)\ \lim_{x\to -1^-} g(x)$ $\quad c)\ \lim_{x\to -1^+} g(x)$ $\quad d)\ \lim_{x\to 2} g(x)$

$e)\ \lim_{x\to \infty} g(x)$

8. $f(x) = \begin{cases} \dfrac{x^2 - x - 20}{x^2 + 8x + 16} & x < 5 \\ \ln(x - 4) & 5 \le x \end{cases}$

List all discontinuities of f(x) and for each a) describe graphically and b) tell what fails in the definition of continuity.

9. Evaluate each limit as infinity, minus infinity or a number.

$a)\ \lim\limits_{x \to \infty} \dfrac{2e^x + 7}{3e^x - 9} \qquad b)\ \lim\limits_{x \to -\infty} \dfrac{2e^x + 7}{3e^x - 9}$

$c)\ \lim\limits_{x \to -\infty} \dfrac{3x^4 + 2}{4x} \qquad d)\ \lim\limits_{x \to \infty} \dfrac{2x^3 + 3x}{4x^4 - 90}$

10. Find all values of x at which f is not differentiable and describe each as a vertical tangent, corner or discontinuity.

$$f(x) = \begin{cases} x^{2/3} & x < 1 \\ x^2 - 6x + 6 & 1 \le x < 4 \\ 16x^{1/2} - 2x - 26 & 4 \le x \end{cases}$$

11. The tangent line to f(x) at x=3 is y= -2x+1. Find f(3) and f '(3).

12. Find the tangent line at the given x-value.

$$a) \quad f(x) = 6x^{1/3} - 2x + 1 \quad at \ x = 8$$

$$b) \quad g(x) = \frac{(x-4)^2}{\sqrt{x}} + 1 \quad at \quad x = 4$$