In Class Exam 3 Review Solutions J. Lewis
Math 142

1 a) $u = x^3 + 5$
$du = 3x^2\,dx$
$\frac{1}{3}\,du = x^2\,dx$

Replace
$\int \underbrace{x^2}_{\frac{1}{3}} \underbrace{(x^3+5)^4}_{u^4} \underbrace{dx}_{du}$

$= \frac{1}{3}\cdot\frac{1}{5}u^5 + C = \boxed{\frac{1}{15}(x^3+5)^5 + C}$

b) $\int (x^6 + 2x^4 + 7x^3 + 14x)\,dx$

$= \boxed{\frac{1}{7}x^7 + \frac{2}{5}x^5 + \frac{7}{4}x^4 + 7x^2 + C}$

c) $u = x^3 + x^2 + 20$
$du = (3x^2 + 2x)\,dx$
$4\,du = (12x^2 + 8x)\,dx$

Replace
$\int 4\,\frac{1}{u}\,du$

$= 4\ln|u| + C = \boxed{4\ln|x^3+x^2+20| + C}$

d) $\int \frac{x^3}{9x^2} + \frac{20}{9x^2}\,dx = \int \frac{1}{9}x + \frac{20}{9}x^{-2}\,dx$

$= \boxed{\frac{1}{18}x^2 - \frac{20}{9}x^{-1} + C}$

e) $u = x-2 \rightarrow x = u+2$

$du = dx \qquad x+5 = u+7$

$$\int (u+7)^2 u^{1/2}\,du = \int (u^2+14u+49)u^{1/2}\,du$$

$$= \int \left(u^{5/2} + 14u^{3/2} + 49u^{1/2}\right) du$$

$$= \frac{2}{7}u^{7/2} + \frac{28}{5}u^{5/2} + \frac{98}{3}u^{3/2} + C$$

$$= \boxed{\frac{2}{7}(x-2)^{7/2} + \frac{28}{5}(x-2)^{5/2} + \frac{98}{3}(x-2)^{3/2} + C}$$

f) $u = x^5 + 5x^2 + 12$

$du = (5x^4 + 10x)\,dx$

$\frac{1}{5}du = \frac{5}{5}(x^4 + 2\cancel{x})\,dx$

$$\int \frac{1}{5}e^u\,du = \frac{1}{5}e^u + C = \boxed{\frac{1}{5}e^{x^5+5x^2+12} + C}$$

2. a) $MAP = \left(\frac{P(x)}{x}\right)' = -0.15 - \frac{3000}{x}$

$$\frac{P(x)}{x} = -0.15x - 3000\ln x + C$$

$$P(x) = -0.15x^2 - 3000x\ln x + Cx$$

$$\frac{P(100)}{100} = 55 = -0.15(100) - 3000\ln 100 + C$$

$$55 = -15 - 13815.51 + C$$

$$13885.51 = C$$

$$P(x) = -0.15x^2 - 3000x\ln x + 13885.51x$$

b) $y'(t) = \frac{t}{t^2+1} \xrightarrow{u=t^2+1} y(t) = \frac{1}{2}\ln(t^2+1) + C$

$$6 = y(0) = \frac{1}{2}\ln 1 + C = C$$

$$\boxed{y = \frac{1}{2}\ln(t^2+1) + 6}$$

c) $u = t^2$

$du = 2t\,dt$

$2du = 4t\,dt$

$$\int 2e^u\,du = 2e^u + C = 2e^{t^2} + C$$

$$y(0) = -3 = 2e^0 + C = 2 + C$$

$$-5 = C$$

$$\boxed{y(t) = 2e^{t^2} - 5}$$

3. $\Delta x = \frac{3-1}{5} = \frac{2}{5} = .4$

$$1 \quad 1.4 \quad 1.8 \quad 2.2 \quad 2.6 \quad 3$$

$$L_5 = .4\left[\frac{1}{1} + \frac{1}{1.4} + \frac{1}{1.8} + \frac{1}{2.2} + \frac{1}{2.6}\right]$$

$$\cong 1.2436$$

$$R_5 = .4\left[\frac{1}{1.4} + \frac{1}{1.8} + \frac{1}{2.2} + \frac{1}{2.6} + \frac{1}{3}\right]$$

$$\cong .9769$$

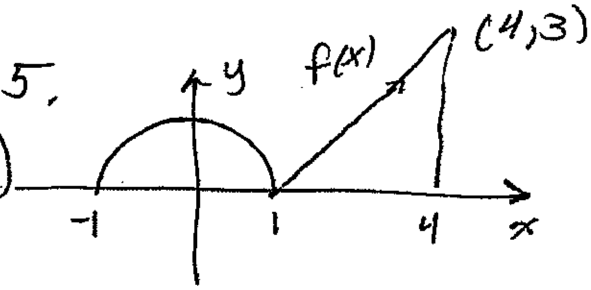
4. $$\frac{d}{dx}\left[xe^x - e^x\right] = e^x + xe^x - e^x$$
$$= xe^x \quad ✓$$

$$\int_a^b xe^x\,dx = \left[xe^x - e^x\right]\Big|_a^b = be^b - e^b - (ae^a - e^a)$$

$$= \boxed{be^b - e^b - ae^a + e^a}$$

5.

a)



$$\int_{-1}^{4} f(x)\,dx = \frac{1}{2}\pi + \frac{1}{2}(4-1)\cdot 3 = \frac{1}{2}\pi + \frac{9}{2}$$

b) $$\int_{-4}^{4} t\sqrt{t^2+1}\,dt = \frac{1}{2}\int_{17}^{17} u^{1/2}\,du = 0$$

$u = t^2+1$ $\quad u(-4)=17$ $\quad u(4)=17$

$du = 2t\,dt$

$\frac{1}{2}\,du = t\,dt$

c) $$\frac{d}{dx}\left[\int_0^x e^{t^2}\,dt\right] = e^{x^2}$$

d) $$\int_0^x \frac{d}{dt}\left(e^{t^2}\right)dt = e^{t^2}\Big|_0^x = e^{x^2} - e^0$$

$$= e^{x^2} - 1$$

$Y1 = 2x^2 - 7x - 12 \qquad Y2 = x^2 - 6x$

6. Math 9 Math NUM enter Y1 - Y2), x, -5, 5)

By hand: Solve $Y1 - Y2 = 0$:

$$2x^2 - 7x - 12 - (x^2 - 6x)$$

$$= x^2 - x - 12$$

$$= (x-4)(x+3) = 0 \text{ at } x=4,\ x=-3$$

$$\left|\int_{-5}^{-3} x^2 - x - 12\, dx\right| + \left|\int_{-3}^{4} x^2 - x - 12\, dx\right| + \left|\int_{4}^{5} x^2 - x - 12\, dx\right|$$

$$= \left|\left[\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 12x\right]_{-5}^{-3}\right| \rightarrow \left|\left(-\tfrac{27}{3} - \tfrac{9}{2} + 36 - \left(-\tfrac{125}{3} - \tfrac{25}{2} + 60\right)\right)\right|$$

$$+ \left|\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 12x\Big|_{-3}^{4}\right| \rightarrow \left|\left(\tfrac{64}{3} - 8 - 48 - \left(-\tfrac{27}{3} - \tfrac{9}{2} + 36\right)\right)\right|$$

$$+ \left|\tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 - 12x\Big|_{4}^{5}\right| \rightarrow \left|\left(\tfrac{125}{3} - \tfrac{25}{2} - \quad - \left(\tfrac{64}{3} - 8 - \quad\right)\right)\right|$$

$$= 16\tfrac{2}{3} + 57\tfrac{1}{6} + 3\tfrac{5}{6} = 77\tfrac{2}{3}$$

6 b) $f(x) = 2x^2 - 7x - 12 \quad g(x) = x^2 - 6x$

Find the interval

solve $f(x) - g(x) = 0$

$$x^2 - x - 12 = 0$$

$$(x-4)(x+3) = 0 \quad x = -3, \; x = 4$$

$$\left| \int_{-3}^{4} x^2 - x - 12 \, dx \right| = \frac{343}{6} \text{ or } 57\tfrac{1}{6}$$

7. a) Find the eq. pt.

$Y1 = 46 - .02x^2 \qquad Y2 = 10 + .02x^2$

2nd TRACE 5 $\quad (\bar{x}, \bar{p}) = (30, 28)$

$$C.S. = \int_0^{30} Y1 - 28 \, dx = \$360$$

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Y1 − 28, x, 0, 30)

$$P.S. = \int_0^{30} 28 - Y2 \, dx = \$360$$

7b) $Y1 = 46 - .01x^2$ $\quad Y2 = 10 + .03x^2$

Window

$X_{MIN} = 0$

$X_{MAX} = \sqrt{4600}$ or guess

$Y_{MIN} = 0$

$Y_{MAX} = 46$

2nd TRACE 5

$(\bar{x}, \bar{p}) = (30, 37)$

equilibrium pt.

$$C.S. = \int_0^{30} Y1 - 37\,dx = \$180$$

$$P.S. = \int_0^{30} 37 - Y2\,dx = \$540$$

Additional example:

Given $\left(\frac{C(x)}{x}\right)' = 0.2 - \frac{500}{x^2}$ = Marginal avg cost

and given avg. cost/unit for 50 units is \$25,

find AC(x) and C(x).

$$\text{Find } (AC) = \frac{C(x)}{x} = \int .2 - \frac{500}{x^2}\,dx = .2x + \frac{500}{x} + c$$

$$\frac{C(50)}{50} \overset{\text{given}}{=} 25 = .2(50) + \frac{500}{50} + c$$

$$= 10 + 10 + c \qquad c = 5$$

$$(AC)(x) = .2x + \frac{500}{x} + 5$$

$$C(x) = .2x^2 + 500 + 5x$$

Find the avg. value of C(x) over [100, 150].

$$\frac{1}{150-100}\int_{100}^{150} C(x)\,dx = \$4291.67$$