Math 142 Quiz 1 Print Name Key 10 pts

1. A manufacturer of a product has fixed costs totaling $8295. Each unit costs an additional $47 to produce. He finds he can never sell any of the product unless the price is below $97, and for each decrease of $5 in the price, he is able to sell 100 more units.

a) Find the cost function.

1 pt $C(x) = 47x + 8295$

b) Find the demand and revenue functions.

1pt $p = d(x) = -\frac{1}{20}x + 97 = -.05x + 97$

$R(x) = -.05x^2 + 97x$

c) Find the profit function.

1 pt $P(x) = -.05x^2 + 50x - 8295$

d) At what two quantities will he break even?

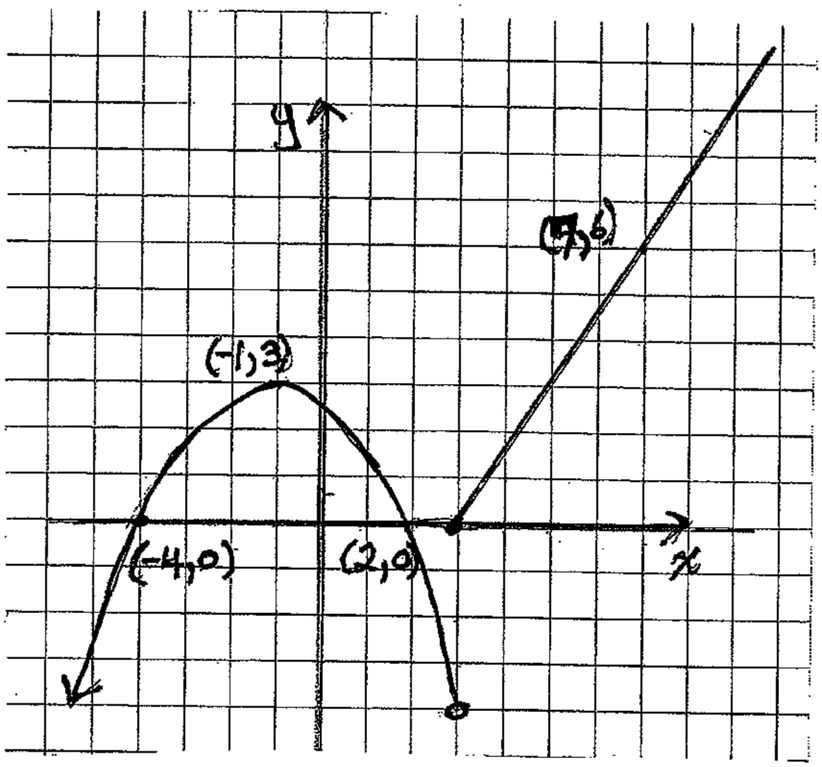
1pt 210, 790

e) At what quantity will he have maximum profit? What is the price at this quantity?

1pt $x = 500$

$p = \$72$

2. The graph shows a quadratic on the left and a line on the right. Determine the formula definition of this piecewise function.



2pts

$$f(x) = \begin{cases} -\frac{1}{3}(x+1)^2 + 3 & x < 3 \\ +\frac{3}{2}x - \frac{9}{2} & x \geq 3 \end{cases}$$

Equivalent forms for the quadratic

$$-\frac{1}{3}x^2 + \frac{2}{3}x + \frac{8}{3}$$

$$-\frac{1}{3}(x+4)(x-2)$$

2pts

3. Find the formula and sketch a graph for each function given that f(x) is exponential and the graph passes through the two given points. Your graph should contain two points, not necessarily the ones given.

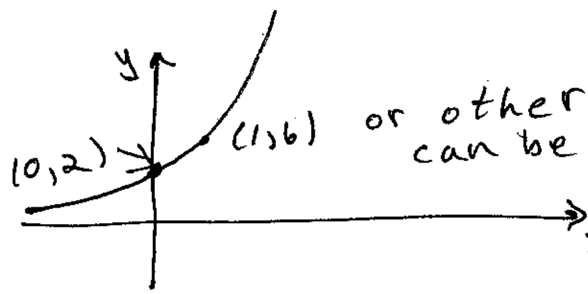
a) f(2) = 18 and f(4) = 162.

$ab^2 = 18 \quad ab^4 = 162 \quad$ so $\quad \frac{ab^4}{ab^2} = \frac{162}{18} = 9$

$b^2 = 9 \quad \boxed{b = 3}$

$a \cdot 9 = 18$ so $a = 2$

$f(x) = 2(3^x)$



or other pts can be shown

b) f(0) = 4 and f(2) = $\overset{0(.16)}{0.016}$. Be careful of the placement of the decimal point.
0.016 = 4/25

0.16

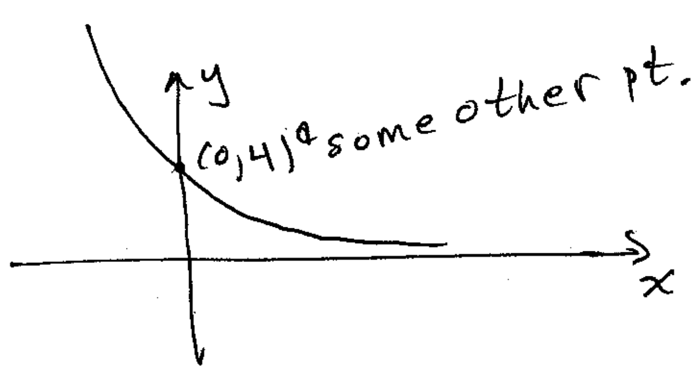
Using 0.16: $a = 4 \quad ab^2 = .16 \quad$ so $b^2 = .04 \quad b = .2$

$f(x) = 4((.2)^x)$

Using 0.016 $\quad a = 4 \quad ab^2 = .016 \quad b^2 = .004 \quad b = \sqrt{.004}$

$4(.004)^{x/2}$ or $4(.06325)^x$ estimate

They can do either one of those.



4. Each set of data is either from a linear function or an exponential function. Find the formula for each.

1pt

a)

x	-1	0	2	5
$f(x)$	13	12.5	11.5	10

$f(x) = -\frac{1}{2}x + 12.5$

b)

x	1	2	3
$f(x)$	9	4.5	2.25

$f(x) = 18(.5)^x = 18\left(\frac{1}{2}\right)^x$

Quiz 2 Print Name Key 10 pts
Math 142 Section or Class time

1. The amount of $1000 is invested at a continuously compounded interest rate, r. After 3 years the value of the account is $1161.84. Find r as a decimal rounded to 4 places. How long will it take the $1000 amount to double? quadruple?

$$\frac{1000e^{3r}}{1000} = \frac{1161.84}{1000} \qquad e^{3r} = 1.16184$$

$$r = \frac{1}{3}\ln(1.16184)$$

2pts

$$= .05 \text{ rounded or } 5\%$$

$$\frac{\ln 2}{.05} = 13.863 \text{ yrs to double or} \sim 14 \text{ yrs}$$

$$\frac{\ln 4}{.05} = 27.7259 \text{ to quadruple or} \sim 28 \text{ yrs}$$

2. Solve for x if

a) $\frac{1}{5}\log_2(x-2)^5 + \log_2(x-4) = 3$.

$$\log_2[(x-2)(x-4)] = 3$$

2pts

$$x^2 - 6x + 8 = 2^3 = 8$$

$$x^2 - 6x = 0 \qquad x(x-6) = 0$$

$\cancel{x=0}$ can't use 0 or $\boxed{x=6}$

b) $\ln(2+x) - \ln(2-x) = 1$

$$\ln\frac{2+x}{2-x} = 1 \qquad \frac{2+x}{2-x} = e \qquad 2+x = e(2-x)$$

$$2+x = 2e - ex$$

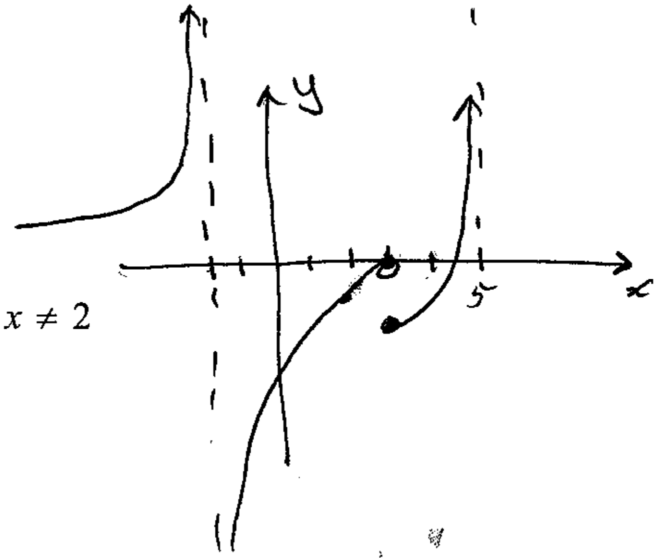
$$ex + x = 2e - 2$$

$$(1+e)x = 2e - 2$$

$$\boxed{x = \frac{2e-2}{1+e}}$$

2pts 3. a) Graph the piecewise defined function,

$$f(x) = \begin{cases} -1 & x = 2 \\ 4\left(\dfrac{x^2 - 5x + 6}{x^2 - 4}\right) & x < 3 \quad x \neq 2 \\ \ln \dfrac{1}{5 - x} & 3 \leq x < 5 \end{cases}$$



Find each limit or state DNE.

a) $\lim\limits_{x \to -2^-} f(x)$ $\lim\limits_{x \to -2^+} f(x)$

$+\infty$ $-\infty$ or DNE

b) Does $\lim\limits_{x \to -2} f(x)$ exist? Is f continuous at -2?

No NO

c) $\lim\limits_{x \to 2} f(x)$ Is f continuous at 2?

-1 yes

$d)\ \lim_{x\to 3^-} f(x)$ $\qquad$ $e)\ \lim_{x\to 3^+} f(x)$

0 $\qquad$ $\ln\left(\frac{1}{2}\right) = -.69...$

Does $\lim_{x\to 3} f(x)$ exist? $\qquad$ Is f continuous at 3?

NO $\qquad$ NO

$e)\ \lim_{x\to 5^-} f(x)$ $\qquad$ ~~Is f left continuous at 5?~~ Don't grade

$+\infty$ $\qquad$ or No

or DNE

2pts 4. $f(x) = \begin{cases} \ln(x^2 - 2x + 2) + 4e^{x-1} & x < 1 \\ 6x + a & 1 \le x \end{cases}$

$$\lim_{x\to 1^-} f(x) = \ln(1-2+2) + 4e^{1-1} = 4$$

What value of a makes f continuous at x=1?

$$\lim_{x\to 1^+} f(x) = 6 + a$$

so make

$6 + a = 4$

$\boxed{a = -2}$

2 pts

5. $f(x) = \dfrac{2}{1 - e^{x}}$

Evaluate each limit as a number, as infinity or as minus infinity, or state DNE.

a) $\lim_{x \to \infty} f(x)$ = 0

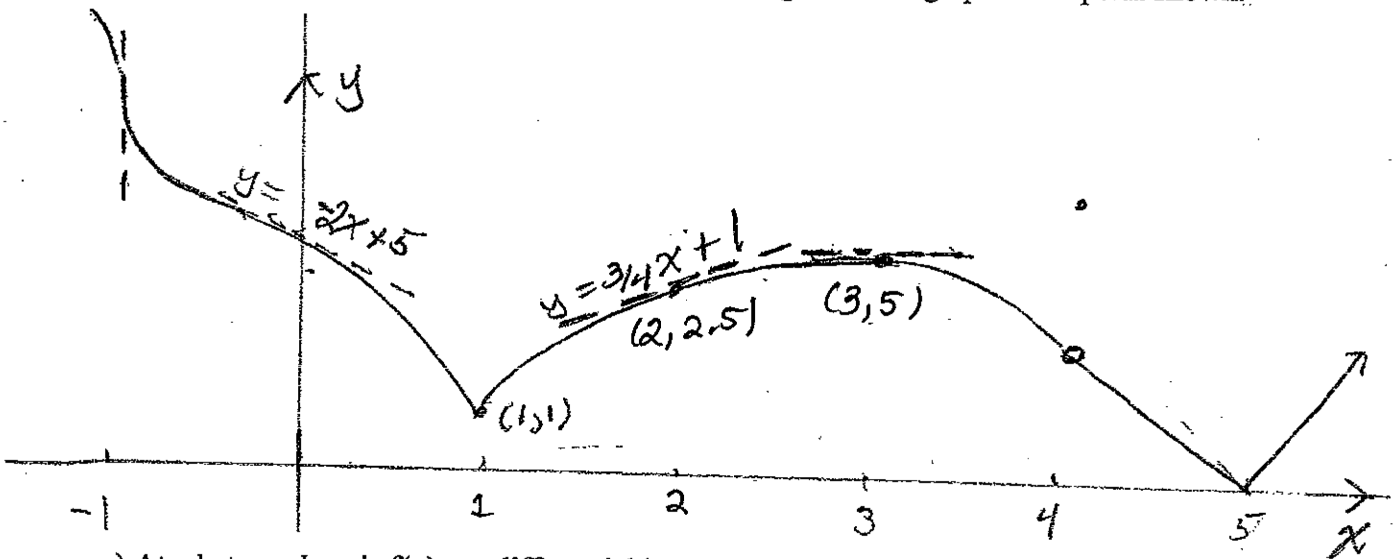
b) $\lim_{x \to -\infty} f(x)$ = 2

c) $\lim_{x \to 0^-} f(x)$ = $+\infty$

d) $\lim_{x \to 0^+} f(x)$ = $-\infty$

Math 142 Fall 2011 Take Home Quiz 3 Print Name Key
Help is Legal Class time

1. The graph of f(x) is shown. Each dotted line is tangent to the graph at the point shown.



a) At what x-values is f(x) not differentiable?

$x = -1$ vertical tangent $x = 4$ discontinuity
$x = 1$ cusp $x = 5$ corner

b) Find the average rate of change of f(x) over the interval [1, 3].

$$\frac{f(3)-f(1)}{3-1} = \frac{5-1}{3-1} = 2$$

c) Find f '(0), f '(2), and f '(3).

$f'(0) = -2$ = slope of tangent line at $x=0$
$f'(2) = 3/4$
$f'(3) = 0$ horizontal tangent, slope $=0$

2. Find the equation of the tangent line to $f(x) = x^{2/3}$ at $x = 8$.
Graph *f(x)* and this tangent line in the same graph.

Pt. of tangency $= (8, 8^{2/3}) = (8, 4)$
Slope $= f'(8)$
Find $f'(x)$ using power rule. Then plug in $x=8$.
$f'(x) = \frac{2}{3}x^{-1/3}$ $f'(8) = \frac{2}{3}(8^{-1/3}) = 1/3$

Use pt.-slope with $m = 1/3$ pt $= (8, 4)$

$$\boxed{y = \frac{1}{3}(x-8) + 4}$$

3. Find each derivative using only the power rule, the shift rule and the linear property. In each case, determine all values of x where the tangent line is horizontal.

a) $f(x) = (x-1)\sqrt{x} = x^{3/2} - x^{1/2}$

$$\boxed{f'(x) = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-1/2}}$$

Horizontal tangent when $\frac{3}{2}x^{1/2} = \frac{1}{2}x^{-1/2}$

$3\sqrt{x} = \frac{1}{\sqrt{x}}$ $\quad 3x = 1$

$x = 1/3$

b) $g(x) = 8(x-1)^5$ is a shift of $8x^5$

$g'(x) = 40(x-1)^4$

$g'(x) = 0$ if $x = 1$ so horizontal tangent at $x = 1$

c) $h(x) = \dfrac{2x^4 - 3x^3 - 12x^2}{x} + 10$

$h(x) = 2x^3 - 3x^2 - 12x + 10$

$h'(x) = 6x^2 - 6x - 12$

Horizontal Tangent:

$6(x^2 - x - 2) = 0$

$6(x-2)(x+1) = 0$ $\quad x = 2$

$x = -1$