Math 142 Take Home Quiz 4 Print Name Key

Help is Legal Section or class time

1. The daily demand quantity, x, for a certain product is 0 if the price per unit is \$35 or more. The quantity demanded changes at the constant rate of 100 more units per \$7 decrease in the price. The fixed cost is \$1200 per day and the total cost increases at the constant rate of \$21 per unit.

a) Find the profit function, P(x).

y-int of demand = (0, 35)

slope of demand = $\frac{-7}{100} = -.07$

$C(x) = 1200 + 21x$

$p(x) = -.07x + 35$ = price/unit

$R(x) = x \cdot p(x)$

$= -.07x^2 + 35x$

$P(x) = -.07x^2 + 35x - (1200 + 21x)$

$\boxed{P(x) = -.07x^2 + 14x - 1200}$

b) Find the exact change in profit if x increases from 75 to 76.

$P(76) - P(75) = -.07(76)^2 + 14(76) - 1200$
$- (-.07(75)^2 + 14(75) - 1200)$

$= +3.43$

c) Use the derivative to find the approximate change in profit if x increases from 75 to 76.

$MP = P'(x) = -.14x + 14$

$P'(75) = +3.50$

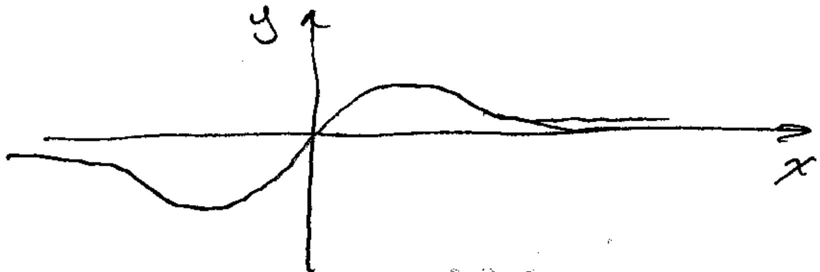
d) Use the derivative to find the approximate change in profit if x increases from 200 to 201. Is profit increasing or decreasing at this production level?

$P'(200) = -.14(200) + 14$

$= -14$ Profit is decreasing by about \$14 per unit.

4. $f(x) = \dfrac{12x}{x^2 + 4}$

a) Graph f(x) in the standard window Zoom 6 and sketch it.



b) Find $f'(x)$. $= \dfrac{12(x^2+4) - 24x^2}{(x^2+4)^2}$

$$= \frac{12x^2+48-24x^2}{(x^2+4)^2}$$

$$= \frac{48-12x^2}{(x^2+4)^2} = \frac{12(4-x^2)}{(x^2+4)^2}$$

c) At what values of x is $f'(x)$ equal to 0? What do you see in the graph at these x values?

$f'(x) = 0$ only if $4 - x^2 = (2-x)(2+x) = 0$

so at $x = -2$ and $x = 2$.

local (relative) min @ $x = -2$

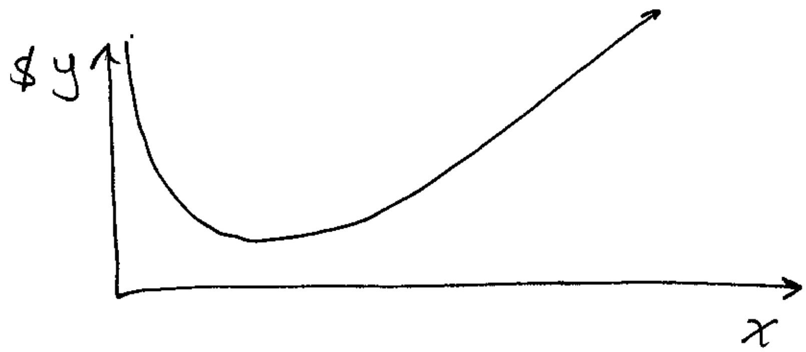
relative max @ $x = 2$

2. A cost function is given by $C(x) = 0.3x^2 + 24x + 2700$ where x is the number of units produced.

a) Find the average cost function.

$$AC = \frac{C(x)}{x} = 0.3x + 24 + \frac{2700}{x}$$

b) Sketch the average cost function after graphing it in your calculator using the window Xmin=0 Xmax=400 Ymin=0 Ymax=200



c) Use 2nd trace and select minimum to find the minimum value of the average cost function. Give both x and y coordinates. What is the value of the marginal cost at this x?

x= 94.87 AC= 80.92 MC= 80.92

$MC = C'(x) = .6x + 24 \qquad C'(94.87) \approx 80.92$

d) Find the marginal average cost function and evaluate it at the x-value of the minimum average cost found in c above.

$$MAC = \left(\frac{C(x)}{x}\right)' = .3 - \frac{2700}{x^2}$$

$$(MAC)(94.87) = .3 - \frac{2700}{(94.87)^2} \approx 0$$

2 pt. bonus

Bonus

e) You should have found the same values for AC and MC in part c above. Use the quotient rule on $\dfrac{C(x)}{x}$ to show that AC and MC are always equal when the derivative of $\dfrac{C(x)}{x}$ is 0.

Using the quotient rule

$$\left(\frac{C(x)}{x}\right)' = \frac{C'(x)\cdot x - C(x)}{x^2}$$

This can be 0 only if $C'(x)\cdot x - C(x) = 0$,

so only if $C'(x)\cdot x = C(x)$

$$C'(x) = \frac{C(x)}{x}$$

$$MC = AC$$

3. Find the derivative of each function.

$a)\quad f(x) = (x+1)^3\sqrt{x-2}$

$$f'(x) = 3(x+1)^2(x-2)^{1/2} + (x+1)^3 \cdot \frac{1}{2}(x-2)^{-1/2}$$

$$= 3(x+1)^2\sqrt{x-2} + \frac{1}{2}(x+1)^3\frac{1}{\sqrt{x-2}}$$

$b)\quad g(x) = \sqrt{x^2+6x}$

$$g'(x) = \frac{1}{2}(x^2+6x)^{-1/2}(2x+6)$$

$$h'(x) = (x^3-8)^{1/2} + x \cdot \frac{1}{2}(x^3-8)^{-1/2} \cdot 3x^2$$

$$= (x^3-8)^{1/2} + \frac{3x^3}{2\sqrt{x^3-8}}$$

$c)\quad h(x) = x\sqrt{x^3-8}$

4. $f(x) = \dfrac{64x}{(x+4)^2}$

a) Find $f'(x)$ and simplify it by factoring $x + 4$ out of the numerator.

$$f'(x) = \frac{64(x+4)^2 - 64x \cdot 2(x+4)}{(x+4)^4}$$

$$= \frac{64(x+4)[x+4-2x]}{(x+4)^{\cancel{4}\ 3}}$$

$$= \frac{64(4-x)}{(x+4)^3}$$

b) Find the equation of the tangent line to $f(x)$ at $x = 0$. Graph $f(x)$ and this tangent line in the window Xmin=-5 Xmax=5 Ymin = -5 Ymax = 5

Pt. of tangency $(0, 0)$ since $f(0)=0$.

Slope $= f'(0) = \dfrac{64 \cdot 4}{(0+4)^3} = 4$

Tangent line $\boxed{y = 4x}$

c) Where is the tangent line horizontal? Find the equation of the tangent line to $f(x)$ at this point. Graph f(x) and the tangent line in the window Xmin=0 Xmax=10 Ymin=0 Ymax = 6

Solve $f'(x)=0$ so $4-x=0$ $\boxed{x=4}$

$(4, f(4)) = (4, 4)$

Tangent line $\boxed{y=4}$

Take Home Quiz 5 Math 142
Help is Legal

Print Name Key
Class time or Section #

2pts or $\frac{1}{2}$ pt. each

1. Find the derivative of each.

a) $f(x) = e^{x^2-3x+7}$

$$f'(x) = (2x-3)e^{x^2-3x+7}$$

b) $f(x) = 3^{\sqrt{x}}$

$$f'(x) = \frac{1}{2}x^{-1/2}\,3^{\sqrt{x}}\ln 3 = \frac{3^{\sqrt{x}}\ln 3}{2\sqrt{x}}$$

either way

c) $f(x) = \ln\left[\frac{(x^4+2x)}{(e^x+12)^3}\right] = \ln(x^4+2x) - 3\ln(e^x+12)$

$$f'(x) = \frac{4x^3+2}{x^4+2x} - \frac{3e^x}{e^x+12}$$

d) $f(x) = \frac{e^{5x}}{x^2+7}$

$$f'(x) = \frac{5e^{5x}(x^2+7) - 2xe^{5x}}{(x^2+7)^2}$$

2 pts
1 each

3. $f(x) = (x+2)^2(x-1)^3$.

a) Find and simplify $f'(x)$. List the x-values where the tangent line is horizontal.

$$f'(x) = 2(x+2)(x-1)^3 + 3(x+2)^2(x-1)^2$$

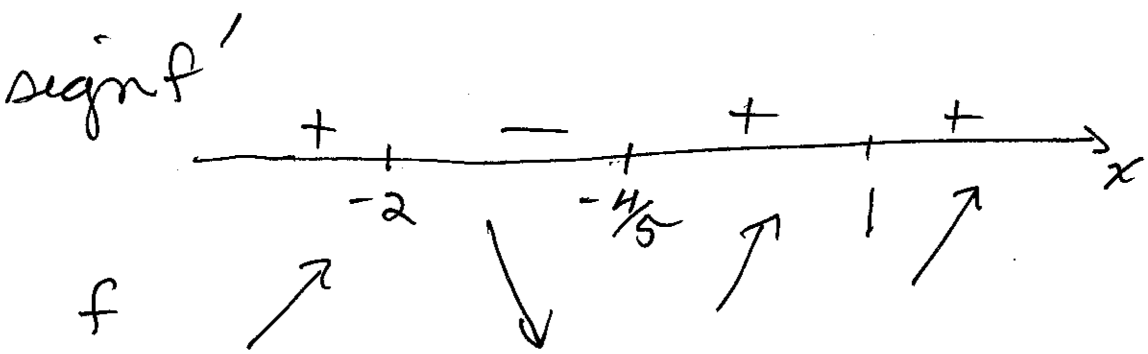
$$= (x+2)(x-1)^2\left[2(x-1) + 3(x+2)\right]$$

$$2x-2+3x+6$$

$$(5x+4)$$

$$= (x+2)(x-1)^2(5x+4)$$

b) Make a sign chart for $f'(x)$. On what intervals is f increasing? decreasing? Locate any local max or min of f.



f is increasing on $(-\infty, -2)$ and on $(-4/5, \infty)$

ok if endpt. is included

f is decreasin on $(-2, -4/5)$

relative max at $(-2, 0)$ or at $x = -2$

relative min at $(-4/5, -8.39808)$ or $x = -4/5$

(They don't have to give the y-value)

2 pts each

4. $f(x) = \dfrac{x}{(x^2+75)^2}$.

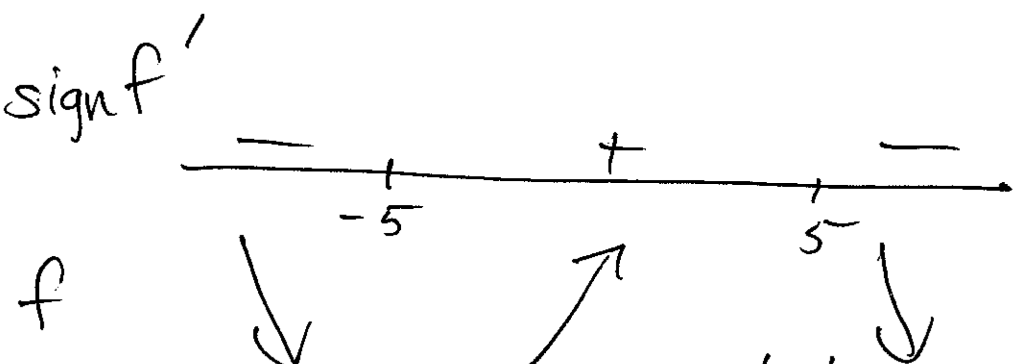
a) Find and simplify $f'(x)$.

$$f'(x) = \left[(x^2+75)^2 - 2x(x^2+75)\cdot 2x\right] / (x^2+75)^4$$

$$= (x^2+75)\left[x^2+75-4x^2\right] / (x^2+75)^4$$

$$= \frac{75-3x^2}{(x^2+75)^3}$$

b) Make a sign chart for $f'(x)$ and find the intervals on which f is increasing and on which f is decreasing. Locate any local max or min of f.



relative min @ $x=-5$

relative max @ $x=5$

f is increasing on $(-5, 5)$

f is decreasing on $(-\infty, -5)$ and on $(5, \infty)$

2 pts

5. Courtesy Barnett, Ziegler and Byleen. The cost per hour for fuel to run a train is $\frac{v^2}{4}$ dollars where v is the velocity of the train in miles per hour. Other costs total $300/hour. How fast should the train travel on a 360 mile trip to minimize the total cost for the trip? Hint: The total number of hours for the trip is distance/velocity = 360/v.

$$c(v) = \left(\frac{v^2}{4} + 300\right)\frac{360}{v}$$

$$= 90v + \frac{108000}{v}$$

$$c'(v) = 90 - \frac{108000}{v^2}$$

$$0 = 90 - \frac{108000}{v^2}$$

if $v^2 = \frac{108000}{90}$

or $v = \sqrt{1200} \approx 34.64$ mph