Marginal- [ Cost, Revenue, and Profit]  

Average Cost  
Average Revenue and Average Profit functions  
Marginal Average Cost  
Marginal Average Revenue and Marginal Average Profit

Introduction:
Recall the limit definition of the derivative
\[
\lim_{h \to 0} \frac{f(x + h) - f(x)}{h} = f'(x)
\]

If this limit exists and \( h \) is "small enough" then
\[
\frac{f(x + h) - f(x)}{h} \approx f'(x) \quad f(x + h) - f(x) \approx h f'(x)
\]

\( f(x+h)-f(x) \) is the exact change in \( f \) and \( hf'(x) \) is the approximate change in \( f \).

If \( x \) is large, say in the thousands as in production quantities, then \( h=1 \) is relatively small. In that case \( f(x+1) - f(x) \approx 1 f'(x) = f'(x) \). This is used to marginal cost, revenue and profit.

The marginal cost function is \( C'(x) \), the marginal revenue function is \( R'(x) \), and the marginal profit function is \( P'(x) \).

Example: Given the profit function for producing and selling \( x \) units is
\[
P(x) = -0.05 x^2 + 150 x - 1500
\]
a) Find the exact change in profit if the production level increases from 2000 to 2001.
b) Use the marginal profit function to approximate the change in profit if \( x \) increases from 2000 to 2001.

a) The exact change in profit is \( P(2001) - P(2000) = -50.05 \). The profit decreases by $50.05. (We get this by substituting 2001 in to P and 2000 into P and subtracting).

b) To approximate this change, first find the marginal profit function, \( P'(x) \). Then substitute \( x=2000 \).

\[
P'(x) = -0.10x + 150 \quad P'(2000) = -50 \quad \text{which is only off by 5 cents.}
\]
This means that profit is decreasing by approximately $50 per additional unit when 2000 units are produced.

Average Cost, \( \frac{C(x)}{x} \)  
Average Revenue, \( \frac{R(x)}{x} \)  
and Average Profit, \( \frac{P(x)}{x} \)

\( \frac{C(x)}{x} \) is total cost / #units = average cost per unit. Similarly for the others.

Marginal Average Cost is \( \frac{d}{dx} \left( \frac{C(x)}{x} \right) \) = the derivative of the average cost function.
Similarly for the others.
Example: Problem 10 pg 206 Section 3.7 in the text book by *Barnett, Ziegler and Byleen*

The total profit from the sale of $x$ charcoal grills is $P(x) = -0.02 x^2 + 20x - 320$

A. Find the average profit per grill if 40 grills are produced.

$$\frac{P(40)}{40} = \frac{-0.02 (40^2) + 20(40) - 320}{40} = 11.20$$

B. Find the marginal average profit at production level 40.
First find the average profit function, then take its derivative, then substitute $x=40$.

$$\frac{P(x)}{x} = \frac{-0.02 x^2 + 20x - 320}{x} = -0.02 x + 20 - 320 x^{-1}$$

$$\frac{d}{dx} \left( \frac{P(x)}{x} \right) = -0.02 + 320 x^{-2}$$

Plug in $x = 40$ to get $-0.02 + 320 / 1600 = -0.02 + 0.2 = 0.18$

What does it mean? The average profit per grill is increasing by approximately 18 cents per additional grill when 40 grills are produced.

A common mistake is to find the average marginal profit. This is not used. **Always find the average (cost, revenue or profit) first and then take the derivative.** Remember: It's the marginal(average cost, average revenue or average profit)