Math 151 Review problems for 5.7-6.2.

1. Find the most general antiderivative of each function

a) \( f(x) = \frac{2}{x^2} - \frac{4}{x} \)  

b) \( f(x) = x^\frac{2}{3} + 2 x^{-\frac{1}{2}} \)  

c) \( f(x) = \sin x - \sqrt{x} \)  

d) \( f(x) = \sec x \tan x \)

2. Find \( F(x) \).

a) \( F'(x) = \frac{1}{x^2 + 1} \) \( F(1) = \pi \)  

b) \( F'(x) = \cos x + \sec^2 x \) \( F\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} \)  

c) \( F''(x) = 2x^3 \) \( F(0) = 2 \) \( F(1) = 4 \)  

b) \( F''(x) = \frac{6}{x^2} \) \( F(1) = 3 \) \( F(e) = 3 \)

e) \( F''(x) = \sin x \) \( F'(0) = 3 \) \( F(0) = 2 \)

3. #80 pg 355 of Stewart. A projectile is fired with an initial speed of 500 m/s at an angle of elevation of 30 degrees from a position 200 m above the ground. Find a) the trajectory of the projectile, b) the maximum height reached and c) the speed when it hits the ground.

4. #70 pg 355 Stewart. A car is traveling at 50 mi/hr when the brakes are fully applied, producing a constant deceleration of 40 ft/s/s. What is the distance covered before the car comes to a stop?

5. #72 pg 355 of Stewart. A car brakes with a constant deceleration of 40 ft/s/s producing, skid marks measuring 160 ft. before coming to a stop. How fast was the car traveling when the brakes were first applied?

6. #82 pg 359 Stewart. In an automobile race along a straight road, car A passed car B twice. Prove that at some time during the race their accelerations were equal.

Chapter 6, 6.1-6.3

7. Evaluate each expression.

a) \( \sum_{k=1}^{20} (2k + 3) \)  

b) \( \sum_{k=1}^{30} (k + 5)^2 \)  

c) \( \sum_{k=1}^{30} (k + 5)^2 \)  

d) \( \sum_{k=10}^{15} \left(\frac{1}{2}\right)^k \)  

e) \( \sum_{k=1}^{10} \left(\frac{1}{3}\right)^{k-1} \)

8. Find the right and left hand Riemann sums for \( f(x) = 4^x \) on \([0, 2]\) using 4 equal subintervals.

Find the average of these two sums.

9. Find the left hand Riemann sum for \( f(x) = x^2 \) on \([0, 3]\) using 20 equal subintervals.
10. Find the left and right hand Riemann sums for \( f(x) = x \) on \([-1, 4]\) using 12 equal subintervals. Use geometry to find the limit of the Riemann sums as the number of subintervals approaches infinity.

11. Find \( \int_a^b f(x) \, dx \) in each case.

a) \( f(x) = \begin{cases} 5 - 2x & \text{if } x < 1 \\ 2 + x & \text{if } x \geq 1 \end{cases} \) \([a, b] = [0, 2]\)

b) \( f(x) \) of part a above, \( a=2, b=0 \).

c) \( f(x) = \sqrt{16 - x^2} \) \([a, b] = [-4, 4]\)

d) \( f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ 2x - 4 & \text{if } x \geq 1 \end{cases} \) \([a, b] = [0, 2]\)

12. For the function of 11 c above, find the area between \( f(x) \) and the x-axis for \( x \) between 0 and 2.

13. Given that \( \int_0^1 f(x) \, dx = -2, \int_1^3 f(x) \, dx = 9, \int_0^3 g(x) \, dx = 4 \), find each of the following:

a) \( \int_0^3 [x + f(x)] \, dx \)

b) \( \int_0^3 [4f(x) - 5g(x)] \, dx \)