Examples of Volumes by Slices

Find the volume of each solid.

1. The base of a solid is the region bounded by $y = x^2$ and $y = 4$. Cross sections perpendicular to the y-axis are squares.

Graph the base.

The square at level $y$ has side $\sqrt{y} - (-\sqrt{y}) = 2\sqrt{y}$ so its area is $A(y) = (2\sqrt{y})^2 = 4y$

Then $V = \int_0^4 4y \, dy = 2y^2 \bigg|_0^4 = 32 \text{ cubic units}$
2. The base is bounded by $y=x^2$ and $y=4$. Cross sections perpendicular to the $x$-axis are squares.

The side of the square at $x$ is $4-x^2$, so its area is $(4-x^2)^2$.

\[ V = \int_{-2}^{2} A(x) \, dx = \int_{-2}^{2} (4-x^2)^2 \, dx \]

\[ = \int_{-2}^{2} 16 - 8x^2 + x^4 \, dx = \frac{512}{15} \]